On the effect of fatigue crack plastic dissipation on the stress intensity factor

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Abstract In metals, during plastic strain, a significant part of the plastic energy is converted into heat. This generates a heterogeneous temperature field around the crack tip which depends on the intensity of the heat source associated with the plasticity and the thermal boundary conditions of the cracked structure under cyclic loading. Due to the thermal expansion of the material, the temperature gradient near the crack tip creates thermal stresses which contribute to stress field around the crack tip. This paper shows how this thermal effect modifies the mode one stress intensity factor for two cases: (i) the theoretical problem of an infinite plate with a semi-infinite through crack and (ii) a finite plate specimen with a central through crack. The comparison of the two cases allows the authors to discuss the effect of convection. The comparison of the simulated and experimental temperature field variation at the specimen surface (infra-red measurement on a mild steel) leads to identify the heat flux in the reverse cyclic plastic zone. This is the key parameter of the problem. Finally, the consequences of the calculation on the range, the ratio and the maximum and the minimum values of the stress intensity factor are discussed.

Keywords: stress intensity factor, plastic dissipation, reverse cyclic plastic zone, thermal stress

1. Introduction

During experimental study of fatigue crack propagation (for example the characterization of the propagation velocity versus the range of the stress intensity factor) the heating effects associated with the crack propagation are often neglected and the tests are considered as isothermal. This assumption is all the more legitimate when the loading frequency is small. However currently it is increasingly necessary to study the fatigue behavior of materials for long and very long life. Experimental techniques of accelerated tests thus are often carry out in order to reduce test durations : electromagnetic resonance fatigue testing machine with a loading frequency of about hundred Hertz and even ultrasonic fatigue machine with a loading frequency of about several tens of kHz. It is then necessary to know if the assumption of an isothermal process is always valid under these test conditions.

In metals, during plastic strain, a significant part of the plastic energy (around 90% [1,2]) is converted in heat. During a cyclic loading of a cracked structure, the plasticity is located in the reverse cyclic plastic zone near the crack tip [3,4]. This heat source generates a heterogeneous temperature field which depends on the intensity of the heat source associated with the plasticity and the thermal boundary conditions of the cracked structure. Due to the thermal expansion of the material, the temperature gradient near the crack tip creates thermal stresses which contribute to the stress field in this region and on the global stress intensity factor.
The objective of this communication is to propose a method in order to quantify the thermal contribution on the stress intensity factor. In a first part the identification of the heat source associated with the crack propagation will be detailed. In a second part the estimation of the thermal effect on the stress field near the crack tip and on the stress intensity factor will be made for the two geometries: an infinite plate with a semi-infinite through crack and a finite plate specimen with a central through crack and with convection boundary conditions on all the specimen faces. Finally, in the last part, all these results will be compared and discussed.

2. Heat source identification associated with the fatigue crack propagation

In order to identify the heat source associated with the crack propagation a cracked specimen which geometry is given in fig. 1, is subjected to a cyclic loading with a stress intensity factor range of about $20\,MPa\sqrt{m}$ and a stress ratio of 0.1. The loading frequency is about 100Hz and the material is a C40 mild steel with an ultimate tensile strength UTS=600 MPa. The mechanical and thermal properties of this steel are summarized in table 1.
The temperature field at the specimen surface was measured with an infrared camera (CEDIP Jade III MWR) whose spectral range is in the near infrared domain. The acquisition frequency and the aperture time of the camera were respectively 5Hz and 1100µs. In order to reduce the effect of the emissivity of the surface on the temperature determination, the specimen was covered with a fine coat of mat black paint.

Table 1. Mechanical and thermal properties of C40 steel

Material properties	density	Yield stress	Young Modulus	Poisson ratio	Thermal expansion	Heat capacity	Heat conduction	Heat diffusivity
Notation	ρ	σ_y	E	ν	α	C	k	a
Unit	kgm^{-3}	MPa	GPa		K^{-1}	$JK^{-1}kg^{-1}$	$WK^{-1}m^{-1}$	m^2s^{-1}
Value	7800	350	210	0.29	$1.2x10^{-5}$	460	52	$1.4x10^{-5}$

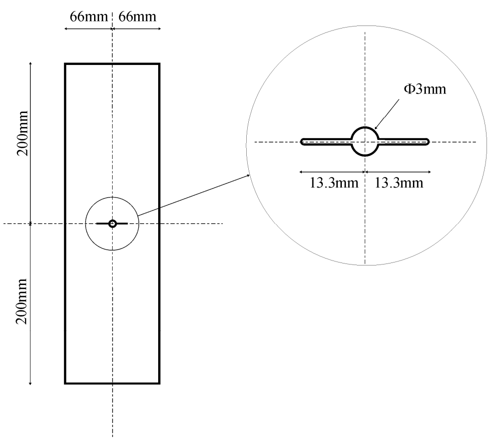

Figure 1. Cracked specimen geometry

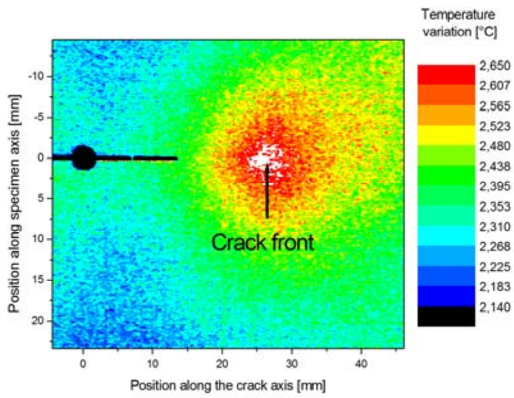

Figure 2. Temperature field near the crack front after 680s

Fig. 2 and fig. 3 show respectively the temperature variation field near the crack tip at time 680s and the temperature evolution according to time at a distance of 5mm from crack front. The temperature evolution on fig. 3 exhibits a superposition of a high frequency evolution of the temperature due to the thermoelasticity and a low frequency signal evolution corresponding to the dissipated power in the reverse cyclic plastic zone. Fig. 2 shows that the temperature distribution remains heterogeneous due to the highly localized form of the heat source in the reverse cyclic plastic zone. The total increase of the temperature in the specimen between the beginning and the end of the test corresponds to the heat source related to the plastic dissipation in the reverse cyclic plastic zone. This total increase in the temperature at a distance of 5mm from the crack front is about 2.5°C.

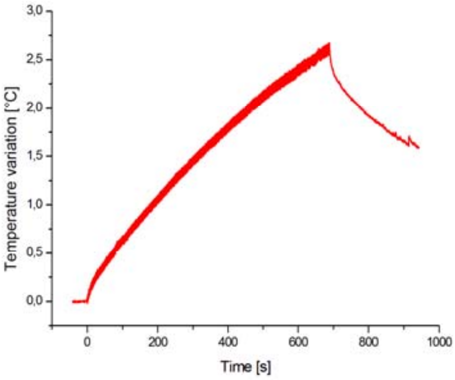

Figure 3. Temperature evolution versus time at a distance of 5mm from the crack front

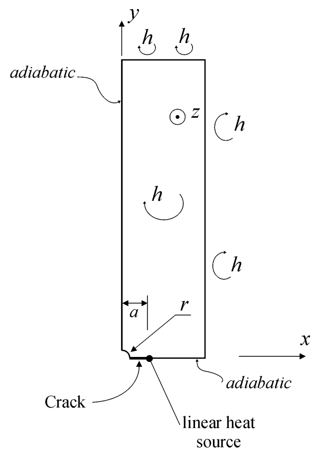

Figure 4. Thermal model, geometry and boundary conditions

In order to determine the heat source associated with the crack propagation. Under plane stress condition the radius of the reverse cyclic plastic zone can be quantified with the relation:

$$r_R = \frac{\Delta K^2}{8\pi\sigma_y^2} \tag{1}$$

For a stress intensity factor of $20\,MPa\sqrt{m}$ the radius of the reverse cyclic plastic zone is about 130µm. This value remains small compared to the size of the specimen. The heat source distribution will be thus considered as linear and centered in the reverse cyclic plastic zone.

For a slow moving crack, the heat source associated with the fatigue crack propagation can be considered to be motionless. This assumption can be justified by the calculation of the Péclet number, noted *Pe*, which compares the characteristic time of thermal diffusion with the characteristic time associated to the heat source velocity (i.e. the velocity of the reverse cyclic plastic zone at the crack tip). In our case the Péclet number is expressed by $Pe = L\mathrm{v}/a$ where *L* is the characteristic length of crack propagation, v the crack velocity and *a* the thermal diffusivity. For a crack length of around 1mm, a crack velocity of 0.1mms^{-1} and a thermal diffusivity of 1.4×10^{-5} m^{2}s^{-1} the Péclet number is 6×10^{-3}. This value remains small compared to unit and therefore the heat source can also be considered as motionless.

In order to identify the heat source a thermal model of the plate was made and solved with the finit element method. The geometry and the thermal boundary conditions are detailed in fig. 4. The heat convection coefficient on the specimen faces and the room temperature are respectively taken equal to 10Wm^{-2}K^{-1} and 20°C. For C40 mild steel, the thermal and mechanical properties are given in table 1. A unit line heat source (q=1Wm^{-1}) is imposed on the crack front line. For a steady state regime the temperature variation at a distance of 5mm of the crack front is about 0.0163°C. Thanks to the linearity of the heat equation with the heat source, it is possible to deduce a heat source of 153Wm^{-1} associated with the fatigue crack loaded with a stress intensity factor range of $20\,MPa\sqrt{m}$.

3. The stress field and stress intensity factor due to the heterogeneous temperature field near the crack tip

2.2. An infinite plate with a semi-infinite through crack

This section is focused on the theoretical problem of an infinite plate with a semi-infinite through crack loaded in fatigue in mode I. The main advantage of this problem is that it can be solved analytically. In the associated thermal problem, the thermal losses due to convection and radiation are neglected and the steady state regime is only considered. This problem is axi-symetric and the temperature variation distribution $\vartheta(r,t)$ is given by the heat equation:

$$\rho C \frac{\partial \vartheta}{\partial t} = q\delta(r) + k \frac{\partial^2 \vartheta}{\partial r^2}, \tag{2}$$

with δ the Dirac function. A solution of this equation is given in [5]:

$$\vartheta(r,t) = \frac{-q}{4\pi k} \mathrm{Ei}\left(-\frac{r^2}{4at}\right), \tag{3}$$

with $-\mathrm{Ei}(-x) = \int_x^\infty \frac{\mathrm{e}^{-u}}{u} \mathrm{d}u$ the integral exponential function and a the heat diffusivity.

In order to estimate the thermal stresses the thermo-mechanical problem with the temperature variation field previously calculated needs to be solved. The behavior of the material is considered elastic and perfect plastic. It is supposed that the plastic strain occurs only in the reverse cyclic plastic zone. With alternating plasticity, the boundary condition on the reverse cyclic plastic zone radius is radial stress equal to zero.

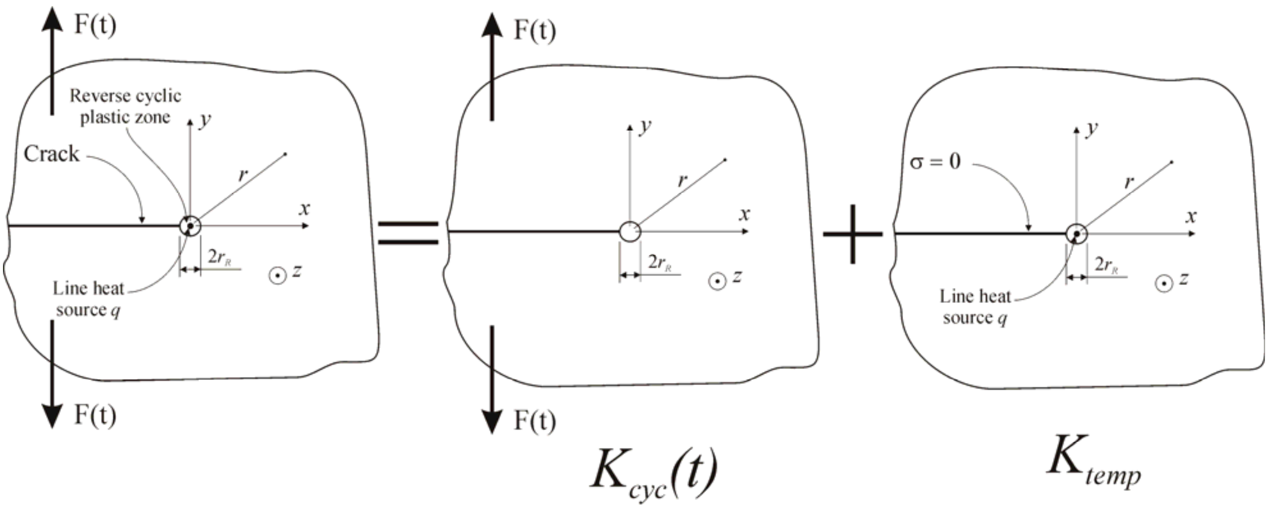

Figure 5. Decomposition of the thermomechanical problem

First the thermomechanical problem can be decomposed into two problems: the first problem (purely mechanical problem) is the cracked specimen subjected only to the cyclic loading $F(t)=F_m+F_a \sin(2\pi f\ t)$ without heat source due to the crack. The stress field associated with this problem is related to a mode one stress intensity factor $K_{cyc}(t)$. The second problem (purely thermal problem) is the cracked specimen subject to the line heat source q. The thermal stresses associated with this thermal loading create to a stress intensity factor named K_{temp}. The thermal effect generates a compressive stress field near the crack front and thus creates a negative contribution on the stress intensity factor ($K_{temp} < 0$) [6]. This decomposition is correct if crack closure due to the thermal effect is neglected and if the thermal effect does not affect significantly the reverse cyclic plastic zone radius. This assumption is realistic if the thermal correction remains small compared to the mechanical loading. The first pure mechanical problem is solved in a classical way and enable us to estimate the mode I stress intensity factor $K_{cyc}(t)$ according to the applied force $F(t)$. In order to solve the second problem, another decomposition is necessary. This second decomposition is illustrated in fig. 6.

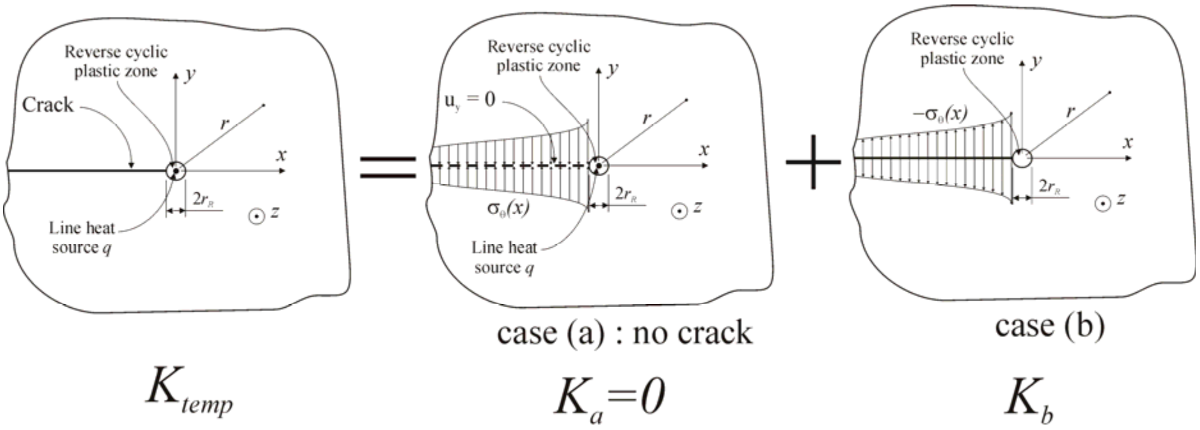

Figure 6. Decomposition of the thermal problem

In the first case (a) a normal stress $\sigma_\theta(r)$ is applied on the crack lips in order to imposed a crack opening equal to zero ($u_y = 0$). This stress $\sigma_\theta(r)$ is calculated from a thermoelastic problem without crack and with the heat source q. The stress intensity factor of case (a) is equal to zero and the stress intensity factor of case (b) is calculated with the Green function:

$$K_{temp} = \sqrt{\frac{2}{\pi}} \int_{r_R}^{\infty} \frac{\sigma_\theta(r,t)}{\sqrt{r - r_R}} \mathrm{d}r \,. \tag{4}$$

The case (a) thermo-mechanical problem is symmetric because both the geometry and the temperature field are symmetric. Only the case of plane stress is considered hereafter but plane strain solution is given in [6]. Further, outside of the reverse cyclic plastic zone since the constitutive behavior of the material is supposed to be elastic, it is expected in first approximation that the basic equations of thermo-elasticity will govern. The equilibrium equation is:

$$r \frac{\partial \sigma_r}{\partial r} + \sigma_r - \sigma_\theta = 0 \tag{5}$$

For which σ_r is the radial normal stress and σ_θ is the circumferential normal stress.
The isotropic elastic stress strain law gives with plane stress hypothesis:

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = \frac{\sigma_r}{E} - \nu \frac{\sigma_\theta}{E} + \alpha \vartheta(r,t) \tag{6}$$

$$\varepsilon_\theta = \frac{u_r}{r} = \frac{\sigma_\theta}{E} - \nu \frac{\sigma_r}{E} + \alpha \vartheta(r,t) \tag{7}$$

A solution of this equation is given in [6] and the circumferential stress can be expressed with the following relation:

$$\sigma_\theta(r,t) = \frac{-\alpha E q}{8\pi k r^2} \left\{ 4at \left[\exp\left(\frac{-\mathrm{r_R^2}}{4\mathrm{at}} \right) - \exp\left(\frac{-\mathrm{r}^2}{4\mathrm{at}} \right) \right] - \left[r_R^2 \mathrm{Ei}\left(\frac{\mathrm{r_R^2}}{4\mathrm{at}} \right) + r^2 \mathrm{Ei}\left(\frac{r^2}{4at} \right) \right] \right\} \tag{8}$$

This circumferential normal stress is then calculated for a line heat source of 153 Wm^{-1} and at time $t = 680$s and this is represented in fig. 7. Near the reverse cyclic plastic zone ($r = r_R = 130$µm) the circumferential stress is negative (about -8.24 *MPa*) because the temperature is high and through the circumferential direction, the material is under compression due to the thermal expansion and the constraint effect.

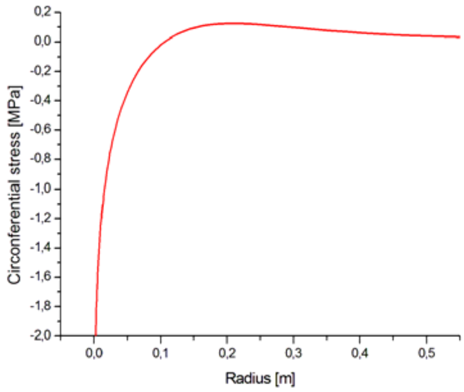

Figure 7. Circumferential stress distribution along radial axis

From equation (1) and (8) it is possible to expressed the associated stress intensity factor K_{temp}. We obtain after integration:

$$K_{temp} = \frac{-\alpha E q}{80k}\sqrt{\frac{2}{\pi}}\left\{ 40\sqrt{r_R} + \frac{20at\left(\exp\left(\frac{-r_R^2}{4at}\right)-1\right)}{r_R^{3/2}} + \sqrt{r_R}\,\mathrm{Ei}\left(\frac{-r_R^2}{4at}\right) \right.$$

$$+ \frac{10(at)^{1/4}}{\Gamma\left(\frac{7}{4}\right)}\left(3\,{}_2F_2\left[\left(\frac{-1}{4},\frac{1}{4}\right),\left(\frac{1}{2},\frac{3}{4}\right),\frac{-r_R^2}{4at}\right]\right) + \frac{10(at)^{1/4}}{\Gamma\left(\frac{7}{4}\right)}\left({}_2F_2\left[\left(\frac{1}{4},\frac{3}{4}\right),\left(\frac{1}{2},\frac{7}{4}\right),\frac{-r_R^2}{4at}\right]\right)$$

$$\left. - \frac{8r_R}{(at)^{1/4}\Gamma\left(\frac{1}{4}\right)}\left(5\,{}_2F_2\left[\left(\frac{1}{4},\frac{3}{4}\right),\left(\frac{5}{4},\frac{3}{2}\right),\frac{-r_R^2}{4at}\right]\right) - \frac{8r_R}{(at)^{1/4}\Gamma\left(\frac{1}{4}\right)}\left({}_2F_2\left[\left(\frac{3}{4},\frac{5}{4}\right),\left(\frac{3}{2},\frac{9}{4}\right),\frac{-r_R^2}{4at}\right]\right)\right\}, \quad (9)$$

With the hypergeometric function

$${}_pF_q = \left(\{a_1,\ldots,a_p\};\{b_1,\ldots,b_q\};z\right) = \sum_{i=1}^{+\infty}\frac{(a_1)_i\ldots(a_p)_i}{(b_1)_i\ldots(b_q)_i}\frac{z^i}{i\,!},$$

where $(a)_i = \frac{\Gamma(a+i)}{\Gamma(a)} = a(a+1)(a+2)\ldots(a+i-1)$ is the Pochhammer symbol and $\Gamma(x) = \int_0^{+\infty} u^{x-1}e^{-x}du$ the Euler Gamma function.

The value of K_{temp}, the thermal correction on the stress intensity factor, is estimated for a line heat source of 153Wm^{-1} and the typical material characteristics detailed in table 1. Eq. (9) gives a thermal correction on the stress intensity factor of -0.521 $MPa\sqrt{m}$.

2.2. A finite plate with a central through crack

For being representative of a real crack propagating problem let us now consider a finite plate with a central through crack. In such case thermal losses due to convection on the specimen faces, the effect of the temperature gradient near the crack front and thus the thermal correction of the stress

intensity factor cannot be neglected. For this type of thermomechanical problem it is not possible to find an analytical solution. That is the reason why computations of both the temperature field and the associated stresses and strains have been carried out by a finite element analysis.
First, the temperature field is calculated with the same model presented in section 2 for the thermal source identification. The steady state temperature field is thus calculated and represented on figure 8 for a line heat source of 153Wm^{-1}.

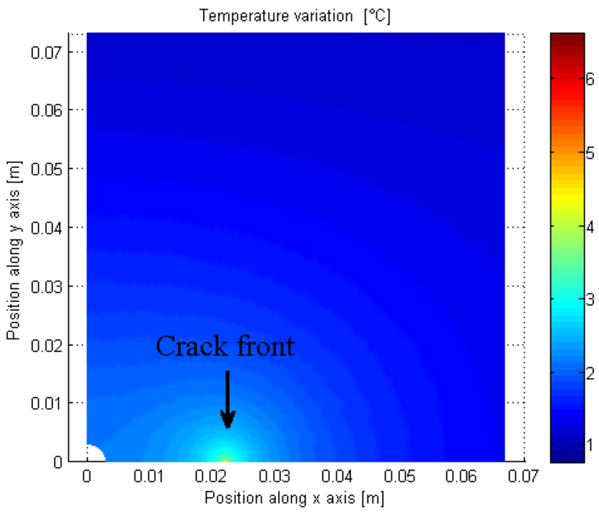

Figure 8. Temperature field near the crack front

The same type of decomposition and assumptions as presented for the previous geometry are used to calculate the stress field in the specimen. These two decompositions are detailed in figure 9 and 10.

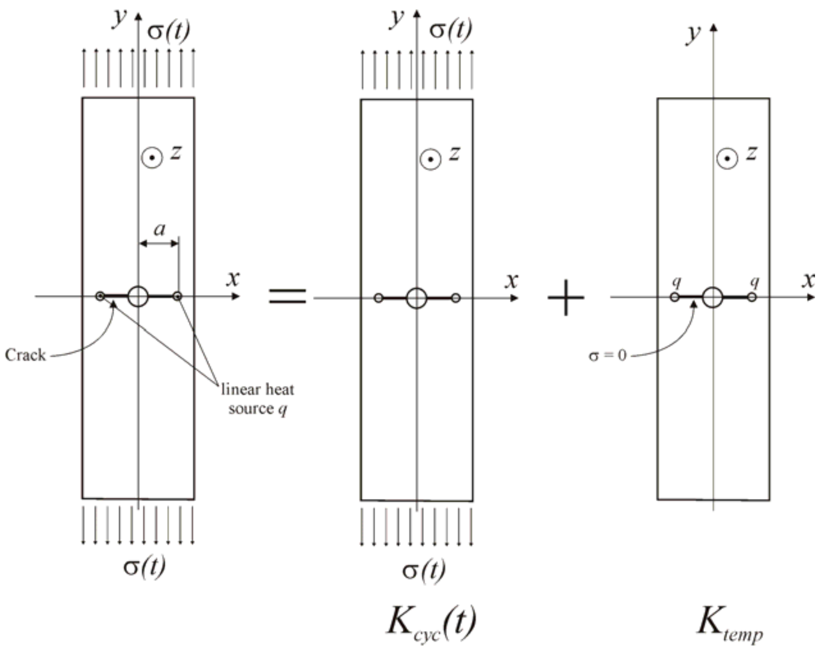

Figure 9. Decomposition of the general problem

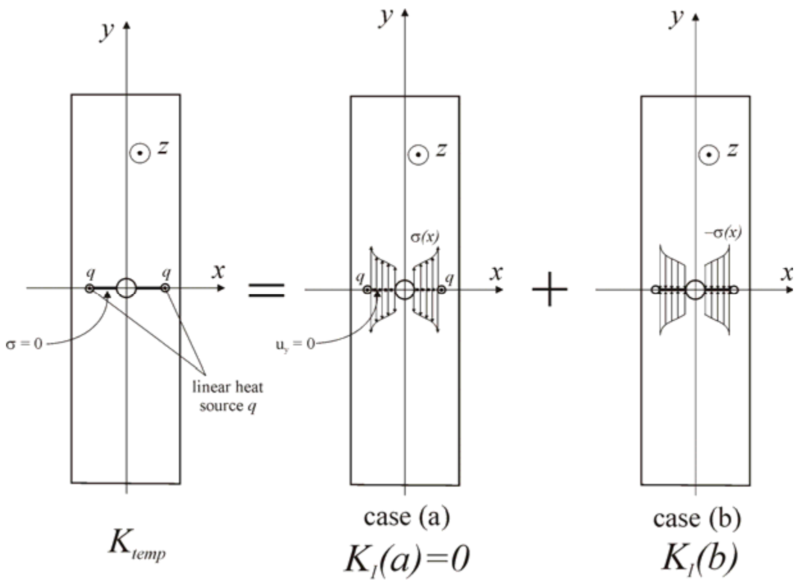

Figure 10. Decomposition of the thermal problem

Figures 11 and 12 show the normal stress in the direction of the y axis. The thermal effect on the stress intensity factor, K_{temp}, is calculated from the case (a) with the Green function and the stress field along the *x* axis calculated with the case (b) :

$$K_{temp} = \frac{2}{\sqrt{\pi}} \int_0^a \sigma(x) \frac{\sqrt{a}}{\sqrt{a^2 - x^2}} \mathrm{d}x \tag{10}$$

with *a* the crack length. The value of the thermal correction of the stress intensity factor K_{temp} for the plane problem with an centered through crack for a line heat source of $153\mathrm{Wm}^{-1}$ is about $-0.316\, MPa\sqrt{m}$.

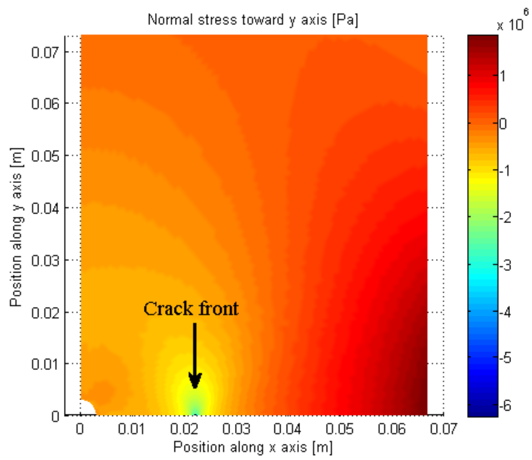

Figure 11. Normal stress field in the direction of the *y* axis

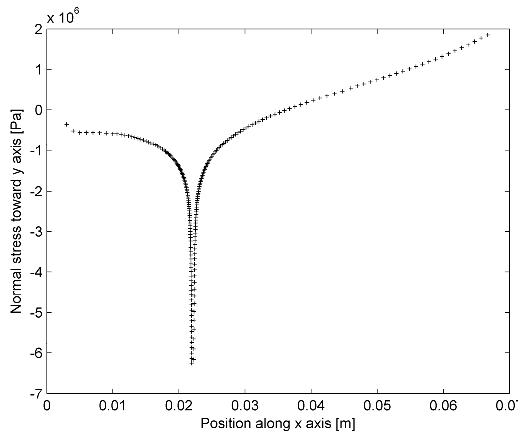

Figure 12. Normal stress in the direction of the y axis distributed along the x axis

2. Conclusion

The correction on the mode one stress intensity factor, K_{temp}, determined in the two previous sections is a value superimposed on the usual stress intensity factor due to the fatigue cyclic loading, noted $K_{cyc}(t)$, which varies at each cycle between a maximum, $K_{cyc,max}$, and a minimum, $K_{cyc,min}$, of the stress intensity factor. As written before, due to the compressive thermal stresses around the crack tip it has been shown that the stress intensity factor during a fatigue loading has to be corrected by the factor K_{temp}. This thermal effect on the stress intensity factor varies slowly with time and can be considered as constant during one cycle. Consequently the temperature has no effect on the stress intensity factor range ΔK but it has an effect on both the maximum, K_{max}, and minimum, K_{min}, values of the stress intensity factor:

$$K_{max}=K_{temp}+K_{cyc,max} \quad (11)$$

$$K_{min}=K_{temp}+K_{cyc,min} \quad (12)$$

However, K_{temp} can affect crack closure by changing the load ratio (equation 13) and because of the compressive nature of the thermal stresses around the crack tip:

$$R_K = \frac{K_{min}}{K_{\max}} = \frac{K_{temp} + K_{cyc,\max}}{K_{temp} + K_{cyc,\min}} \neq \frac{K_{cyc,\max}}{K_{cyc\min}} \quad (13)$$

In the two problem geometries presented in this paper, the results on the thermal correction of the stress intensity factor are very closed. For a stress intensity factor range of $20\,MPa\sqrt{m}$ and a stress ratio of 0.1 the thermal correction is about $-0.521\,MPa\sqrt{m}$ for an infinite plate with a semi-infinite through crack and $-0.316\,MPa\sqrt{m}$ for a finite plate with a central through crack with considering thermal losses due to convection. In conclusion the geometry of the specimen and the thermal boundary conditions have a very small effect on the results. For test on mild steel at a loading frequency of 100Hz, the values of K_{temp} remain very small and a new stress ratio of 0.087 (compared to 0.1, the initial stress ratio) can be calculated.
However, since the dissipated energy rate per unit length of the crack front is proportional to both the loading frequency and to $\Delta K^4/\sigma_y^4$ this effect should be more important for ductile metals (low yield stress) loaded under high stress intensity range. Revisiting the frequency effect on the fatigue crack growth could be also interesting by taking this thermal correction consideration.

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