Analysis and residual strength prediction for multiple site damage in aircraft structures by using weight function method

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Abstract
This paper presents a review of recent progress of the development of weight function method for analyzing multiple site damage in aircraft structural panels. An analytical method, the Weight Function Method (WFM), has been proposed and further developed to analyze the MSD problems and to determine the related various fracture mechanics parameters. The plastic zone sizes and crack opening displacements were extensively verified and validated by excellent agreement with available analytical and finite element method results. Combined with Crack Tip Opening Angle criterion (CTOA), the present WFM is employed to predict the stable crack growth and residual strength of aircraft aluminum sheets containing MSD. The predicted results compared very well with the corresponding experimental data. Thereby a novel analytical approach with high efficiency and reliability has been devised for MSD analyses in aircraft structures.
Keywords: Multiple site damage, Weight function method, Plastic zone size, Crack opening displacement, Residual strength.

1. Introduction

Multiple Site Damage (MSD) has been a serious threat to modern transport airplane structure safety, and therefore has become a great concern in aircraft damage tolerance design and airworthiness certification. A historical milestone case of this type of failure was the in-flight disintegration of a 5.5m long piece of the pressure cabin skin of upper fuselage of Aloha Airlines Boeing 737-200 over Hawaii in 1988 [1]. After the Aloha accident, all the major commercial airplane manufacturers were required to evaluate their aircraft for MSD in the critical areas of the wing, empennage and pressure fuselage [2].

Various models and methods [3-5] have been developed to determine the stress intensity factors, plastic zone sizes and crack opening displacements for the prediction of the fatigue life and residual strength for structures with MSD. Most current approaches rely mainly on advanced numerical techniques, especially the Finite Element Method (FEM). Despite its powerfulness, reliable FEM solutions for MSD require great efforts, time and experience in modeling and computation. Classical analytical method, e.g. the complex variable method, is limited to idealized MSD configurations.

Recently, the Weight Function Method (WFM) has been proposed and further developed by the authors to analyze various fracture mechanics parameters and stable crack growth of sheets with MSD[6-10]. It is shown to be versatile and cost-effective for tackling sheets with MSD problems. The approach is outlined in this paper.

2. Weight function method for fracture parameters analysis for collinear cracks

2.1. Basic principle

According to the weight function theory, for a crack subjected to an arbitrary pressure σ(x) distributed at the crack faces, the non-dimensional stress intensity factor f can be determined by a simple quadrature [11,12].

$$f = \int_0^a \frac{\sigma(x)}{\sigma} \cdot \frac{m(a,x)}{\sqrt{\pi a}} \mathrm{d}x \qquad m(a,x) = \frac{E'}{f_r(a)\sigma\sqrt{\pi a}} \frac{\partial u_r(a,x)}{\partial a} \tag{1}$$

where $m(a, x)$ is the weight function for the crack body, $E'=E$ for plane stress, $E'=E/(1-\nu^2)$ for plane strain, σ is a reference stress, a and x are the non-dimensional crack length and coordinate along the crack normalized by the characteristic length W (often taken as unity), here W refers to half plate width for the finite width panel containing a center crack. f_r(a) and $u_r(a, x)$ are the stress intensity factor and crack opening displacement, respectively, for a reference load case.

The corresponding crack opening displacements can also be easily determined when the relevant weight function, $m(a, x)$, is available. From Eq. (1), we have

$$u(a,x) = \frac{\sigma}{E'} \int_{a_0}^{a} [f(s)\sqrt{\pi a}] \cdot m(s,x) \mathrm{d}s \tag{2}$$

where the non-dimensional stress intensity factor $f(s)$ is obtained using Eq. (1).

It should be emphasized that, the σ(x) in Eq. (1) refers to the stress distribution at the prospective crack line, and is determined from stress analysis for the same configuration but without crack. This implies that once the weight function is known for a given crack geometry and σ(x) is determined, the stress intensity factors and crack opening displacements for any crack length can be obtained by simple integration through Eqs.(1) and (2). The advantages are especially useful for stress intensity factor and strip yield model analysis of multiple site damage.

2.2. Weight function method for special collinear cracks

The weight function method for single crack is applied for a special multiple site damage case[7]: one large center crack formed by coalescing three un-equal length center cracks in a panel of finite width, with compressive yield stress σ_s uniformly acting along the un-cracked ligament and in the crack tip region, Fig.1a.

The analysis for this case is conducted by assuming the coalesced three un-equal length cracks as one single fictitious crack subjected to segment pressure distribution in plastic zones, in addition to the applied external load, Fig.1b. Essentially, the Dugdale[13] strip yield model is the superposition of two linear elastic solutions. One is for remote uniform tension stress, which is available in Ref. [14]. Another is for segment uniform compressive yield stress acting in the plastic zones. The stress intensity factor and crack opening displacement for this load case can be determined by using WFM, equations (1) and (2). The weight functions for a center crack in a finite sheet were given in Ref.[12].

For the three coalesced cracks, the critical stress and the fictitious crack length are determined based on two conditions [4]: i) Vanishing of the stress singularity at the fictitious crack tips shown in Fig.1b; and ii) Zero of the minimum crack opening displacement at the ligament [a_1, a_1+d].

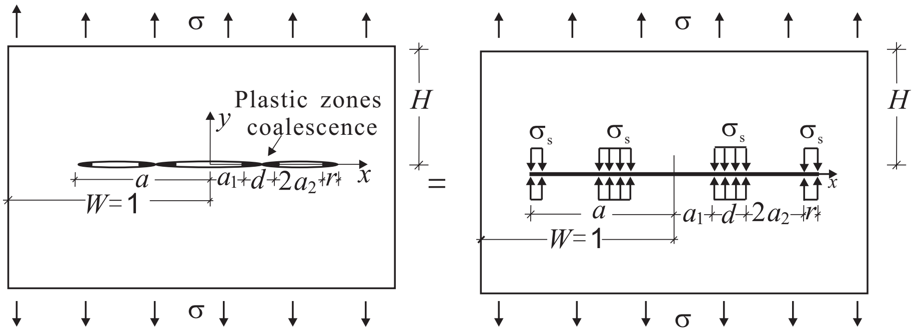


Fig.1 A coalesced center crack in a finite width panel containing three un-equal cracks, the total length of the fictitious crack includes all the strip yield zones

2.3. Weight function method for general collinear cracks

In this section, the weight functions for general collinear cracks are presented. The derivation of the weight function method for general collinear cracks was based on the reciprocity theorem and the superposition principle [8]. It was found that the weight functions for general collinear cracks are quite different from that for a single crack configuration.

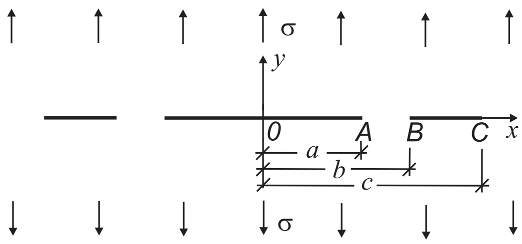


Fig.2. Three symmetric collinear cracks in an infinite sheet subjected to remote uniform stress

Take a typical MSD configuration, three collinear cracks in an infinite sheet, Fig.2, as an example. The weight functions for the crack tips *A*, *B* and *C* are given in Eqs.(3-5), respectively [8].

$$m_a(a,b,c,x)=\frac{E'}{f_{rA}(a,b,c)\cdot\sigma\sqrt{\pi a}}\cdot\begin{cases}\partial u_1^r(a,b,c,x)/\partial a;\ x\in[0,a]\\ \partial u_2^r(a,b,c,x)/\partial a\ ;\ x\in[b,c]\end{cases}\tag{3}$$

$$m_b(a,b,c,x)=\frac{-E'}{f_{rB}(a,b,c)\cdot\sigma\sqrt{\pi(c-b)/2}}\cdot\begin{cases}\partial u_1^r(a,b,c,x)/\partial b\ ;\ x\in[0,a]\\ \partial u_2^r(a,b,c,x)/\partial b\ ;x\in[b,c]\end{cases}\tag{4}$$

$$m_c(a,b,c,x)=\frac{E'}{f_{rC}(a,b,c)\cdot\sigma\sqrt{\pi(c-b)/2}}\cdot\begin{cases}\partial u_1^r(a,b,c,x)/\partial c;\ x\in[0,a]\\ \partial u_2^r(a,b,c,x)/\partial c;x\in[b,c]\end{cases}\tag{5}$$

where the non-dimensional stress intensity factors f_{rA} (a,b,c), f_{rB} (a,b,c) and f_{rC} (a,b,c) for remote uniform tension stress were given in Ref.[14]. The corresponding crack opening displacements for center and side crack $u_1^r(a,b,c,x)$ and $u_2^r(a,b,c,x)$ are:

$$u_1^r(a,b,c,x)=\frac{2\sigma}{E'}\int_{-a}^{x}\frac{x\left\{x^2-\left[\left(c^2-a^2\right)E(k)/K(k)+a^2\right]\right\}}{\sqrt{\left(a^2-x^2\right)\left(b^2-x^2\right)\left(c^2-x^2\right)}}\cdot\mathrm{d}x\ ;\ -a\le x\le a\tag{6a}$$

$$u_2^r\left(a,b,c,x\right)=\frac{2\sigma}{E'}\int_b^x\frac{x\left\{x^2-\left[\left(c^2-a^2\right)E\left(k\right)/K\left(k\right)+a^2\right]\right\}}{\sqrt{\left(x^2-a^2\right)\left(x^2-b^2\right)\left(c^2-x^2\right)}}\mathrm{d}x\;;\;b\le x\le c \tag{6b}$$

Having obtained the weight functions for this crack configuration, the stress intensity factors, crack opening displacements and plastic zone size of its strip yield model can be determined [8].

2.4. A unified method for strip yield collinear cracks

It is observed that the weight function method is accurate and efficient to obtain the stress intensity factor and crack opening displacement. However it is hard to obtain the weight functions for cracks in finite sheet. In this section, a simple and efficient unified method [9] is proposed to solve the strip yield model for collinear cracks in infinite and finite sheets. The key idea of this method is to treat all the cracks, the plastic zones and the remaining elastic ligament between the cracks as a single crack, which is solved using the weight function method. For example, the strip yield model for two collinear cracks in an infinite sheet shown in Fig.3a is modeled by an equivalent single center crack shown in Fig.3b. The center crack of length $2l=(4a+2b+2r_B)$ is subjected to (i) remote uniform stress σ; (ii) segments uniform compression yield stress $-\sigma_s$ over the plastic zones r_A and r_B; and (iii) continuous compression stress $-\sigma(x)$ distributed along the remaining elastic ligament $x\in[-(b-r_A), (b-r_A)]$, respectively. The $\sigma(x)$ is further discretized by a set of uniform segment stress, as shown in Fig.3c. The requirements for the equivalent single crack are: 1）No stress singularity at the fictitious crack tip, 2) Zero crack opening displacements along the elastic ligament between the fictitious crack tips, $x\in[-(b-r_A), (b-r_A)]$.

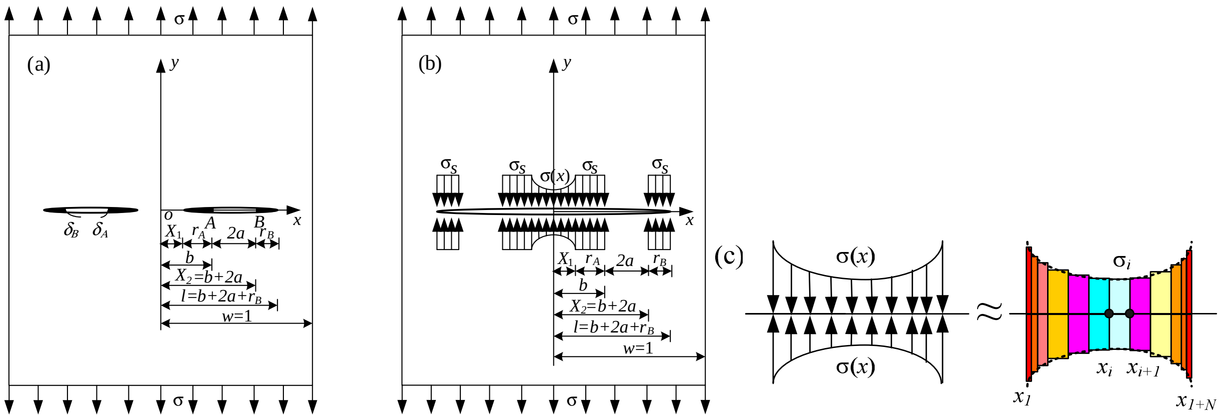


Fig.3. The concept of the unified method: (a) a strip yield model for two collinear cracks in an infinite sheet, (b) an equivalent single crack for modeling problem (a), (c) discretized stress distribution.

The stress intensity factor F and crack opening displacement $U(l, x)$ for the equivalent crack subjected to such complex loading are determined by the superposition of three component elastic solutions. Closed form solutions for the first two load cases are available in Ref.[12]. For the equivalent crack, the following equations are established to obtain the unknown variables.

$$\begin{cases}F=0\;;\\U\left(l,x\right)=0\;;\;x=\left(x_i+x_{i+1}\right)/2,b-r_A,-\left(b-r_A\right);i=1,2,3\ldots N\end{cases} \tag{10}$$

Having established the equations, the plastic zone sizes and stress distribution along the elastic ligament $\sigma(x)$ can be determined by solving the above equation using the weight function method and Newton-iteration. Then, the corresponding crack opening profile for the strip yield model can be obtained by superposition.

2.5. Examples and validations

In this section, two and three collinear cracks in finite and infinite sheets are solved using the above methods. Some existing results and FEM results are also presented for comparison.

2.5.1 Three collinear cracks with plastic zones coalesced in a finite sheet

For a given three crack configuration a_1=0.3, a_2=0.1 and d=0.2 in a finite sheet shown in Fig.1, the critical applied stress $\sigma_c/\ \sigma_s$ and fictitious crack length a are obtained based on the method described in section 2.2, which are 0.3777 and 0.7417 respectively. Figure 4 shows the corresponding CODs determined by WFM and FEM, and very good agreement is observed.

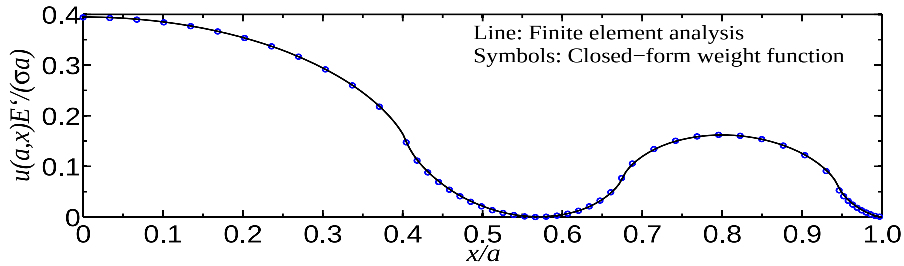


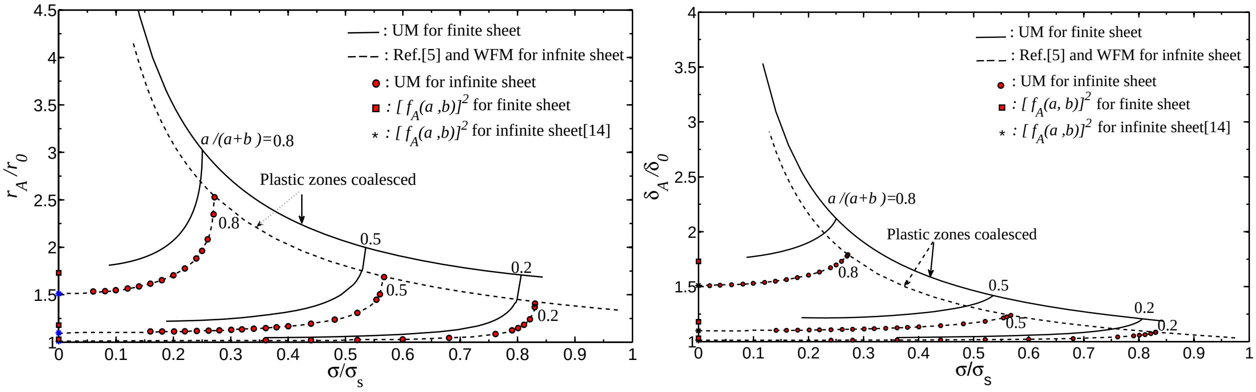
Fig.4. Crack opening profile for the fictitious crack of three un-equal length cracks with coalesced plastic zones. (In Fig.1, a_1 = 0.3, d = 0.2, $2a_2$ = 0.2, the half-length of fictitious crack a = 0.7417) [7].

2.5.2 Two equal-length collinear cracks in finite and infinite sheets

The strip yield models for two equal-length collinear cracks infinite sheets can be obtained by using the WFM and the unified method. However, for the cracks in finite sheet, the unified method is applied. Figure 5 shows some typical results of the inner plastic zone sizes and CTOD as a function of the applied stress. These results are normalized by the plastic zone size r_0 of a single Dugdale crack of the same length, $r_0=a[\sec(0.5\pi\sigma/\sigma_s)-1]$, $a=(c-b)/2$. Also shown in these figures are the results for the plastic zones critical coalescence. To verify the solution accuracy of the present weight function approach, the results are compared to those given by Collins and Cartwright [5] by using complex stress function method. It is observed that the results for two collinear cracks in an infinite sheet obtained from the weight function method, complex stress function method and unified method agree very well. Fig.6 shows the CODs for the strip yield model of the two cracks in finite sheet subjected various applied loads. Also shown in this figure are the FEM results, very good agreement is observed. However, the unified method (UM) is much more efficient than FEM for solving the strip yield model.

2.5.3 Three collinear cracks in an infinite sheet

Figure 5 and 6 show the strip yield model for three symmetric collinear cracks in an infinite sheet. The plastic zone sizes and CODs were obtained by using the WFM and unified method.



(a) Plastic zone size r_A (b) Crack tip opening displacement δ_A

Fig.5 Plastic zone sizes and crack tip opening displacement for two collinear cracks in finite and infinite sheets[9].

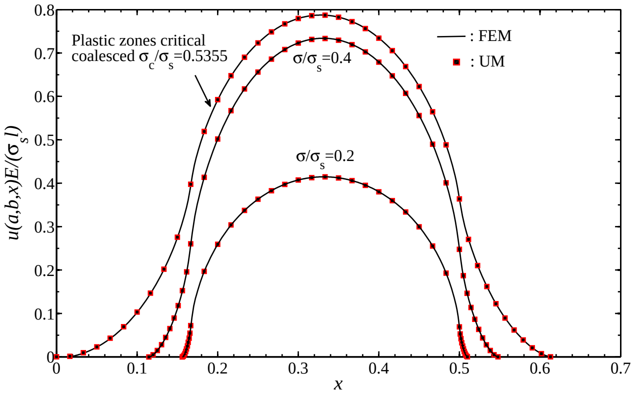


Fig.6 Crack opening profiles for the strip yield model for two equal-length collinear cracks in a finite sheet shown in Fig.4a with a=b=1/6[9].

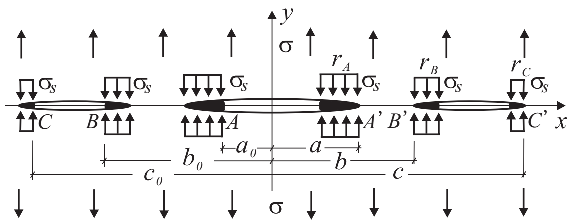
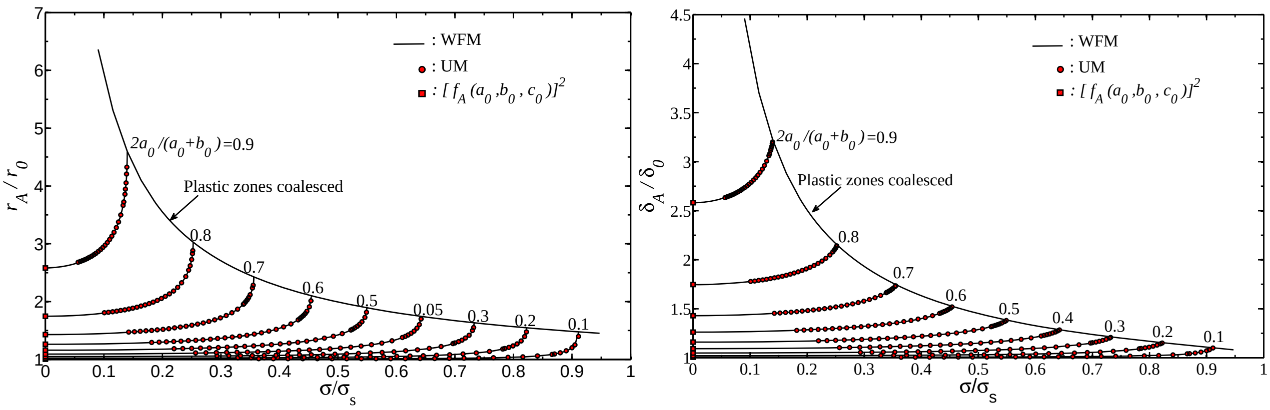


Fig.7. Strip-yield model for three collinear cracks, with separated plastic zones.



(a) Plastic zone size r_A (b) Crack tip opening displacement δ_A

Fig.8. Plastic zone sizes and CTOD for three equal-length collinear cracks in infinite sheets [9]

Some results for the inner crack tip A of three collinear cracks are given in Fig.8. It is observed that the results obtained from the unified method agree well with those from weight function method. It is also found that for $\sigma/\sigma_s \rightarrow 0$, the crack tip plastic zone size and opening displacement for the crack tip A approach the non-dimensional stress intensity factor squares $[f_A(a_0,b_0,c_0)]^2 \cdot r_0$ and $[f_A(a_0,b_0,c_0)]^2 \cdot \delta_0$ shown in Figs.8a and b, respectively. The variables r_0 and δ_0 are the crack tip plastic zone size and opening displacement for a center crack in an infinite sheet.

3. Residual strength prediction and validation for sheets with MSD

3.1 CTOA criterion based on strip yield model

The crack tip opening displacement in combination with the strip yield model had been used by several researchers to predict the stable crack growth behavior [10,15]. In the method, the crack growth was controlled by two parameters. One is critical crack opening displacement δ_0 which is used to describe the crack initiation, Fig.9a; the other is used to characterize the stable crack growth by a constant critical CTOA α, Fig.9b. The crack growth equation for the whole process is

$$\begin{cases} \delta\left(a_0, a_0\right) = \delta_0; \text{crack initiation} \\ \delta\left(a, a-d\right) = 2d\tan\left(\alpha_c/2\right) + \delta_c\left(a-d, a-d\right); \text{crack propagation} \end{cases} \tag{8}$$

where, $\delta(a, a\text{-}d)$ is the crack opening displacement at a distance d behind the crack tip, the first variable in the bracket is the crack length, and the second is the x location. $\delta_c(a\text{-}d, a\text{-}d)$ is the plastic wake height, which is equal to the crack tip opening displacement. α_c is the critical CTOA.

In practice, a pair of 'optimal' δ_0 and α_c is selected as the critical values. Using the 'optimal' values, the predicted crack growth behaviors of coupon specimens agree very well with the corresponding experiment observations. Here, the C(T) specimen is used to determine these critical values. The weight function method is adopted here to determine the COD for crack growth analysis. Figure 10 shows the predicted load-crack extension curves obtained by three different pairs of parameters. Also in the figure are the results measured from experiment [10]. It is observed that the predicted result obtained by the parameters δ_0=0.10mm and α_c=4° agrees well with test data. It is assumed that the criterion for single crack is also applicable to sheets with MSD. And these critical values will be used as material properties to predict crack growth for MSD specimens.

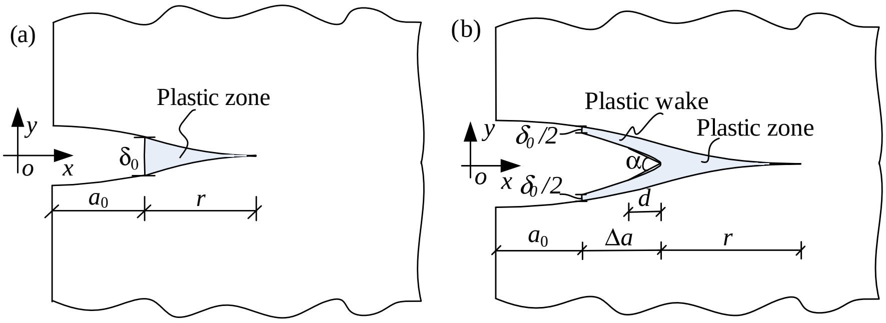


Fig.9. Crack opening profile of modified Dugdale strip yield model at (a) initiation and (b) at propagation with definition of crack growth parameters α and δ_0

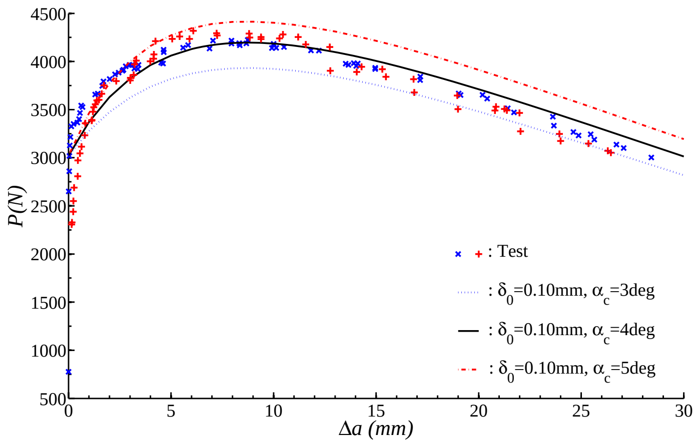


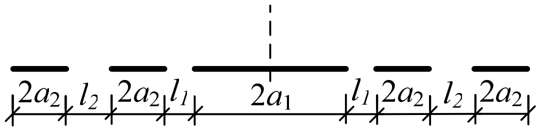
Fig.10 Crack extension against load of C(T) specimen (Δa-P curve)[10]

3.2 Experimental and predicted stable crack growth

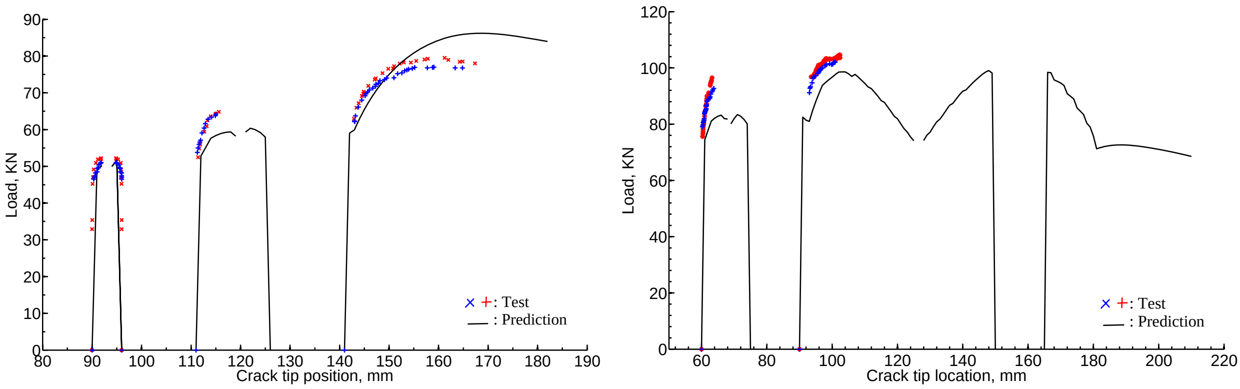
With the critical δ_0 and α_c values determined, the main task for stable crack growth analysis of sheets with MSD is to solve the strip yield model to obtain the crack opening displacement for cracked panels. As a result, the efficiency and accuracy of the stable crack growth prediction are much dependent on the method for solving the strip yield model.

The unified method was applied to determine the COD for stable crack growth analysis. Here, the results for sheets (600mm×1140mm) with five different cracks shown in Fig.11a are presented as examples. Figure 11b and c shows the predicted and experimental crack growth behaviors for two sheets with five different cracks. In both cases, the length of the side cracks is 15mm. The length of the center lead crack and ligaments between cracks are different. In Fig.11b, the experimental and predicted maximum residual strengths occurred after the fracture of all the ligaments. In Fig.11c, the sheet failed immediately at the fracture of ligament l_2 shown. Due to the quick fracture of this specimen, the crack growth information for the outside crack tip was not recorded. However, the predicted result was able to describe the entire crack growth process, see the solid lines in Fig.11. When the applied load reached at the maximum residual strength 99.0KN, all the crack tips started to grow, resulting in the fracture of the whole sheet. More detailed information about the experiment and prediction on various MSD configurations was given in Ref.[10].

The elastic-plastic FEM is widely used to predict the residual strength for cracked structures. Using the “plane strain core” model [16] and CTOD criterion embedded in ABAQUS software, the ligament fracture loads and residual strengths for some of the MSD configurations were given in Ref.[10]. It is found that the accuracy of both methods is comparable. However, the computational and modeling demands are quite different. For a given MSD configuration, it takes at least *two hours* (a computer with a Pentium® Dual-Core CPU E5300@2.60GHz and 3.00GB RAM) to complete a residual strength prediction by using FEM. Yet, it does not include the time for creating the finite element model. In order to model the crack growth, the “debond” technique in ABAQUS was used. The FEM involves material and contact non-linear analysis [10]. Rich experiences on finite element modeling and analysis are required. However, for most of the MSD configuration given in Ref.[10], *two minutes* is enough to complete a residual strength analysis by using the present method. Furthermore, once the crack growth analysis program for a given crack configuration is available, there is no modeling time. These advantages are very useful for parametric analysis.



(a) Schematic geometrical dimensions for five collinear cracks



(b) a_1=90mm, l_1=6mm, l_2=15mm; (c) a_1=60mm, l_1=15mm, l_2=60mm;

Fig.11 Experimental and predicted Δ*a*-*P* curves for sheet with five collinear cracks

4 Conclusions

An analytical approach, the weight function method for dealing with the MSD problems is presented in this paper. The study leads to the following conclusions:

1．For special collinear crack configurations which can be treated as a single crack problem, such as three collinear cracks with strip yield plastic zones critical coalescence in a finite sheet, the strip yield model solutions can be easily solved by the weight functions for a single crack. The results are in perfect agreement with FEM results.

2．Weight function formulas for more general collinear cracks have been derived, which are markedly different from those for the single crack cases. With the derived weight functions, the key fracture mechanics parameters, stress intensity factors and crack opening displacements for the three collinear cracks under arbitrary load conditions are easily computed by a simple quadrature.

3. A unified method based on the weight function for a single crack is proposed to solve the strip yield models for collinear cracks in infinite and finite sheet. The method is used to solve the strip yield models for two and three symmetrical collinear cracks in infinite and finite sheets to obtain plastic zones, crack opening displacements and stress distributions along the elastic ligaments between cracks. These results are widely compared with exact solutions and FEM results, perfect agreements are observed. This method is simple, efficient, reliable, and versatile.

4. Combined with CTOD criterion, the WFM is used to predict the stable crack growth behaviors and residual strengths of MSD configurations in finite-width sheets subjected to monotonic loading. The solution efficiency is significantly better than the FEM.

In summary, the present WFM for MSD provides a powerful analytical approach for fracture mechanics analyses and residual strength assessment for MSD-contained aircraft structures.

References

[1] R.J.H.Wanhill. Milestone case histories in aircraft structural integrity. In: Milne I, Ritchie RO, Karihaloo B, editors. Comprehensive Structural Integrity, Oxford: Elsevier Ltd; 2003, 61-72.

[2] FAA. Aging Airplane Program: Widespread Fatigue Damage. Federal Register Rules and Regulations 2010; 75(219): 69746-69789.

[3] X.Y.Deng, J.W.Hutchinson. Approximate methods for analyzing the growth of large cracks in fatigue damaged aircraft sheet material and lap joints. Finite Elem Anal Des 1996; 23: 101-114.

[4] T. Nishimura. Strip yield analysis of two collinear unequal cracks in an infinite sheet. Engng Frac Mech 2002; 69: 1173-91.

[5] R.A.Collins, D.J.Cartwright. An analytical solution for two equal-length collinear strip yield cracks. Engng Frac Mech 2001; 68: 915-24.

[6] W.Xu, X.R.Wu, H.Wang. Weight functions and strip yield solution for two equal-length collinear cracks in an infinite sheet. Engng Fract Mech 2011; 78(11): 2356-2368.

[7] X.R.Wu, W.Xu. Strip yield crack analysis for multiple site damage in infinite and finite panels – A weight function approach. Engng Fract Mech 2011; 78(14): 2585-2596.

[8] W.Xu, X.R.Wu. Weight functions and strip-yield model analysis for three collinear cracks. Engng Fract Mech 2012; 85(1):73-87.

[9] W.Xu, X.R.Wu, H.Wang. A unified method for strip yield model solution of multiple collinear cracks. Engng Fract Mech 2012. (under review).

[10] W.Xu. Weight function method for aircraft structure multiple site damage analyses and residual strength prediction[PHD thesis]. Shanghai Jiao Tong University.2012. (in chinese).

[11] H.F.Bueckner A novel principle for computation of the stress intensity factors. Z Angew Math Mech 1970; 50(9): 529-546.

[12] X.R.Wu, A.J.Carlsson. Weight functions and stress intensity factor solutions. Oxford: Pergamon Press; 1991.

[13] D.S.Dugdale. Yielding of steel sheets containing slits. J. Mech. Phys. Solids 1960; 8:100-104.

[14] H.Tada, P.C.Paris, G.R.Irwin. The stress analysis of cracks handbook.3rd ed. New York:ASME Press; 2000.

[15] R.Galatolo, KF.Nilsson. An experimental and numerical analysis of residual strength of butt-joints panels with multiple site damage. Engng Fract Mech 2001; 68(13):1437-1461.

[16] J.C.Newman Jr, M.A.James,U.Zerbst. A review of the CTOA/CTOD fracture criterion. Engng Fract Mech 2003;70(3-4): 371-385.