Mesh Dependence of Transverse Cracking in Laminated Stainless Steel with both High Strength and High Ductility

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Abstract A combination of surface mechanical attrition treatment (SMAT) and co-rolling can produce large-scale laminated nanostructured stainless steel (SS) with both high strength and high ductility. Recent numerical results based on the cohesive finite element method have revealed that brittle nanograined interface layer can enhance the ductility of the co-rolled SMATed SS. Here, numerical investigation focuses on effects of both shape of the bilinear cohesive law and mesh size and shows that the larger thickness of the phases in uniform state allows the use of coarser meshes.

Keywords Laminated nanostructured metals, cohesive finite element method, bilinear cohesive law, mesh dependence

1. Introduction
Surface mechanical attrition treatment (SMAT) and co-rolling can be combined to produce large-scale laminated nanostructured metals with both high strength and high ductility [1]. Therefore, it is believed to have a good future to be applied in structural engineering. Through the SMAT, a nano-crystalline surface can be generated for various metals to enhance their yield stress and fatigue life. The co-rolled SMATed metals with nanograined interface layers (NGILs) can be produced. This approach can generate laminated nanostructured 304 stainless steel (SS) with tensile yield stress 878 MPa and failure strain 48% [1].

A computational framework for fracture analysis is in need to investigate the toughening mechanism. The main concern is on the nucleation and the propagation of non-localized microcracks into coarse-grained layer (CGL). Therefore, the cohesive finite element method (CFEM) is appealing to investigate the cracking. The CFEM can model damage initiation/evolution and fracture processes explicitly and has been used to investigate brittle and ductile fracture extensively. The intrinsic CFEM embeds cohesive elements along boundaries of all volumetric elements as part of the physical model [2]. It was adopted in our former studies [3,4]. Our studies have shown that both the critical energy release rate and the thickness of the NGIL play critical roles in determining the overall ductility of the co-rolled SMATed 304SS. However, the dependences of the results on both shape of the cohesive law and the mesh have not been addressed.

2. Mesh dependence issue and numerical framework
Many cohesive laws have been developed [5]. The bilinear cohesive law was used in [3,4], as shown in Fig. 1. $T_{\text{max}}$, cohesive strength, is the stress at which the damage initiates and the...
separation is $\delta^0_m$. The critical energy release rate is given by

$$G_{coh} = \int_0^{\delta_m^f} T(\delta) d\delta = 0.5 T_{max} \delta_m^f,$$

where $T$ is an effective traction, $\delta$ an effective separation, and $\delta_m^f$ a critical crack opening.

The damage associated with the cohesive surface separation can be defined [6]. Many researchers
assumed that $T_{max}$ and $G_{coh}$ played key roles while the shape of the cohesive law was relatively
unimportant. Series of studies indicated that it could lead to unreasonable results [5,7]. Therefore,
the shape of the cohesive law should be considered. Dependence of the results on the shape of
the cohesive law is closely relevant to the dependence of the results on the mesh. Stress distribution
in the cohesive zones must be resolved accurately. Therefore, a minimum number of elements are
needed in each cohesive zone and the minimum number has attracted much attention [8]. On the
other hand, insertion of cohesive elements introduces fictitious compliance [9]. To alleviate it,
Geubelle and Baylor [9] used an adjustable initial slope in the bilinear cohesive law.

The co-rolled SMATed 304SS contains the CGL and the NGIL. A computational configuration with
a length 1 mm and a width 0.9 mm is illustrated in Fig. 2. The thickness of the NGIL is taken as 40
$\mu$m. Two levels of structured cross-triangular mesh size, namely, 10 by 10 $\mu$m and 5 by 5 $\mu$m, are
used. The cross-triangular meshes with uniform size 10 by 10 $\mu$m are illustrated in Fig. 2.
Constitutive parameters for the bulk and cohesive elements are listed in Table 1. Both phases are treated as isotropic, elasto-plastic metals. The density, Young’s modulus $E$, Poisson’s ratio $\nu$, and flow stress for the two phases are the same as those in [3,4]. The critical energy release rate of the CGL, $G_{IC}$, is obtained in terms of its fracture toughness $K_{IC}$, which is taken as 100 MPa$\sqrt{m}$. The cohesive strength of the NGIL can be calibrated at different critical energy release rate and failure strain [3,4]. Two types of boundaries between the CGL and the NGIL are considered: i) a tough boundary implying that its cohesive parameters are the same as those of the CGL and ii) a brittle boundary implying that its cohesive parameters are the same as those of the NGIL. NGIL’s failure strain was estimated to be 3.26% [3]. Here, the critical energy release rate of the NGIL is taken as 60 Jm$^{-2}$ and its cohesive strength is calibrated as 1.89 GPa [3,4]. With the calibrated results, simulations are carried out at two levels of the cohesive strength of the CGL ($T_m=1.87\sigma_0$ and $1.94\sigma_0$) and for two types of boundaries.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Density (kg/m$^3$)</th>
<th>Yield stress $\sigma$</th>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$T_{\text{max}}$</th>
<th>$G_{\text{coh}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CGL</td>
<td>8000</td>
<td>$\sigma_0$</td>
<td>200</td>
<td>0.29</td>
<td>$T_m$</td>
<td>$G_{IC}$</td>
</tr>
<tr>
<td>NGIL</td>
<td>8000</td>
<td>$\sigma_0'$</td>
<td>200</td>
<td>0.29</td>
<td>$T_m'$</td>
<td>$G_{IC}'$</td>
</tr>
</tbody>
</table>

3. Numerical results

3.1 Effect of the shape of bilinear cohesive law

$\delta_m^0$ is taken as $10^2 \mu$m and $10^3 \mu$m. When the mesh size is taken as 10 and 5 $\mu$m, the simulated results for the co-rolled SMATed 304SS are shown in Figs. 3(a-b) and 4(a-b), respectively. With $\delta_m^0$ decreasing, the computational burden increases obviously, consistent with the finding in [10], and the overall fluctuation in the stress-strain curve also decreases. As $\delta_m^0$ decreases, simulations show that the peak stress around the microcrack nucleation decreases in Figs. 5 and 6, which is in agreement with [7].

3.2 Effect of the mesh size

For refined mesh, the incipient microcrack occurs relatively earlier, more minor branches occur and then arrest after running one or a few elements length, similar with the experimental study [11]. On comparing Figs. 3 with 4, it can be observed that the stress history when the mesh size is 5 $\mu$m is more detailed than that when the mesh size is 10 $\mu$m. Meanwhile, at each level of $\delta_m^0$, both the
peak stress before the microcrack nucleation and that after the microcrack nucleation have negligible dependence on the mesh size.

Figure 3. The results with mesh size 10 µm when δ_m^0 is (a) 10^{-2} and (b) 10^{-3} µm

Figure 4. The results with mesh size 5 µm when δ_m^0 is (a) 10^{-2} and (b) 10^{-3} µm

The cohesive zone size has been studied extensively. The widely-used estimate in plane strain condition is Rice’s model [12], which gives the cohesive zone size to be \( \frac{9E}{32}\left(\frac{Hq_L}{f_p}\right)^2 \). For the CGL, the estimated cohesive zone sizes are 7690 and 7190 µm. The mesh sizes, 10 and 5 µm, are much smaller than them. For the NGIL, the estimated cohesive zone size is 3.24 µm, smaller than the mesh sizes. Due to its brittle nature, the size of cohesive zones in the NGILs is relatively easy to reach the level of the NGIL thickness. The cohesive zones in the NGILs contain 4 and 8 cohesive elements when the mesh size is 10 and 5 µm, respectively. Therefore, there is no need to further refine the mesh in the NGILs. The above justifies using two levels of mesh size. Note that in Figs. 3 and 4, at each level of δ_m^0, the main results are fairly consistent despite the fact that the mesh sizes are not apparently smaller than the estimated cohesive zone size for the NGILs. Previous
investigation has also suggested that the mesh size requirement can be relaxed under certain conditions [8,9]. Our simulation shows that the larger thickness of the brittle phase in uniform state allows the use of coarser meshes.

4. Conclusions
The simulations show that the shape of the bilinear cohesive law can obviously change the stress response, including the overall numerical fluctuation and peak stress. For crack initiation and propagation in a laminated composite structure, overall stress response has weak dependence on the mesh size. Simulation results show that the comparatively larger thickness of the phases in uniform state relaxes the requirement on the mesh size.

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