FATIGUE CRACK GROWTH FROM A SURFACE FLAW

Masanori Kawahara* and Masayoshi Kurihara*

INTRODUCTION

Surface flaws or notches have long been recognized as a major origin of potential failure in structural elements. Fatigue crack growth from a surface flaw has been extensively studied for several years as one of the most important subjects for the safe life estimation of structures such as pressure vessels, piping systems, LNG carriers, etc.

Analysis of fatigue crack growth has generally been performed by the use of Paris’ formula which relates crack growth rate to stress intensity factor. This approach has been confirmed by numerous experimental studies on two-dimensional problems such as the growth of an edge-crack in a plate or axially symmetrical crack in a round bar.

Several researchers [1-4] tried to apply the same principle to the surface crack problems using the results of numerical analyses of stress intensity factor of a semi-elliptical crack performed by Smith et al [5], Shah et al [6], Rice et al [7], etc. This approach, however, has only partially been successful: Paris’ formula gives a good interpretation to the experimental combined loads [2,4]. This apparent disagreement may be due to the difficulty in obtaining the exact solution of stress intensity factor in reality or plasto-elasticity.

In the present paper, the authors present a set of experimental results on fatigue crack growth from a surface flaw under axial tension, single bending or combined loads [8], and discuss the possibility of a simplified approach, other than the direct application of Paris’ formula. Analysis of fatigue crack growth was performed separately on the change equation was obtained to estimate the safe life against plate-thickness in design. 

EXPERIMENTAL PROCEDURES

Tests of fatigue crack growth from a surface notch were conducted in two types of high strength low alloy steels, HT-60 and HT-80, and a mild steel, DM-41. Chemical composition and mechanical properties are shown in Table 1.

Three types of specimen were prepared as shown in Fig. 1: T-type for axial tension, B-type for simple bending and TB-type for combined tensile and bending loads. The initial surface notch was made by the so-called “notch-in-rib” method: During the machining of specimen surface, a rib of height...
of 2 mm was left unmachined in the center of plate, and a notch was made in the rib by a saw-cut without harming the rest of plate surface. This method enables us to obtain initial surface notches of a length more than 1 mm.

Fatigue tests were conducted in a 50 ton Amalor type machine at a frequency of 300 cpm. T-type specimens were subjected to pulsating tension. B-type specimens were loaded by 4-point bending. The T8-type specimen was put on a back plate and fixed by high tension bolts and welding at end-edges of plate. This assembly was then subjected to 4-point-bending, which yields a stress as shown in Figure 2.

Fatigue crack growth was principally examined by the "beach mark" method: At appropriate intervals during the fatigue test, the load amplitude was altered to one half of its normal value for 3,000 to 10,000 cycles, while the maximum load was kept constant. This operation causes traces of crack front contour to appear as beach marks on the fracture surface.

Test conditions are summarized in Table 2.

CHANGES IN CRACK SHAPE DURING GROWTH

A part-through surface crack is generally treated as a semi-elliptical crack, which is characterized by crack depth, a, and crack length, b, as shown in Figure 3. In order to analyze the changes in crack shape during growth under various stress conditions, the relationship between the depth-to-length ratio, a/b, and depth-to-thickness ratio, a/h, was examined in each specimen.

Figure 4 shows the a/b vs a/h relation in specimens with a very short initial notch: 2b₀ = 1 mm, which agrees well with the following equation:

\[
a/b = (0.98 + 0.07 R_B) - (0.06 + 0.94 R_B)a/h \tag{1}
\]

where

\[
R_B = \Delta S_T / (\Delta S_T + \Delta S_B)
\]

\[\Delta S_T: \text{range in axial tensile stress}\]

\[\Delta S_B: \text{range in bending stress}\]

For a rough estimation, equation (1) may be simplified as follows:

\[
a/b = 1 - R_Ba/h \tag{1'}
\]

Figure 5 shows the a/b vs a/h relation in a specimen with a longer initial notch: 2b₀ = 10, 20, 40 mm. In this case, the following equation agrees well to experimental results:

\[
a/(b^n - b_0^n) = (0.98 + 0.07 R_B) - (0.06 + 0.94 R_B)a/h \tag{3}
\]

or as a simple expression:

\[
a/(b^n - b_0^n) = 1 - R_Ba/h \tag{3'}
\]

where \( n \) is a constant nearly equal to 2, that may depend on material. All the data were well explained by putting \( n = 2 \), except some data in axial tension, which were influenced by a secondary bending stress.

Equation (3) leads to the following equation which enables us to estimate crack depth, a, putting \( n = 2 \),

\[
a^* = \frac{0.98 + 0.07 R_B}{1/(b^2 - b_0^2)^{1/2} + (0.06 + 0.94 R_B)/h} \tag{4}
\]

Figure 6 shows the comparison between measured crack depth, a, and calculated value, \( a^* \). It may be noted that equation (4) gives a fairly good estimate of crack depth. A simpler equation is obtained from equation (3'), as follows:

\[
a^{**} = \frac{1}{1/(b^2 - b_0^2)^{1/2} + R_B/h} \tag{4'}
\]

CRACK GROWTH RATE

It was tried, as a first step, to apply Paris' formula to the analysis of crack growth rate, using the numerical results on stress intensity factor of semi-elliptical crack calculated by Shah et al [6]. The agreement with experimental results, however, was poor as presented in Figure 7.

A number of empirical expressions, then, were examined in trial and error approach to obtain a best fitting equation for crack growth rate, and finally the following equation was obtained:

\[
db/dN = C [(\Delta S_T + \Delta S_B/2) (nb)^m] \tag{5}
\]

where \( C \) and \( m \) are material constants as in Paris' formula.

Equation (5) gives the crack growth rate in the length direction, \( db/dN \), as a function of half crack length, \( b \), and independent of crack depth, \( a \). Figure 8 shows the confirmation of equation (2), the relation between \( db/dN \) and \( (\Delta S_T + \Delta S_B/2) (nb)^m \). The validity of equation (5) is limited to the data for cracks initiated from a very short initial notch and those from a longer initial notch when the amount of fatigue crack growth exceeds 15% to 20% of the plate thickness in the depth direction.

The crack growth rate in the depth direction is then obtained by the combination of equation (5) and equation (1) or (3), which gives:

\[
da/db = \frac{a^2/b^2}{(0.98 + 0.07 R_B)} \tag{6}
\]

for cracks from a very short initial notch, and

\[
da/db = \frac{a^2/b^2}{(0.98 + 0.07 R_B)(1 - b_0^n/b^n)^{1/n}} \tag{7}
\]
for cracks from a longer initial notch. Equation (7) is reduced to equation (6) when $b_0/b$ is sufficiently smaller than unity.

Combining equation (5) and equation (6), the crack growth rate in the depth direction, $da/dN$, is given as follows, for cracks from a small notch:

$$\frac{da}{dN} = C[(\Delta S_T + \Delta S_B/2)(nb)^{1/2}]^{m} \cdot \frac{a^{2}/b^{2}}{(0.98 + 0.07 R_B)}$$

(8)

A more simpler expression is obtained by combining equation (5) and equation (1'):

$$\frac{da}{dN} = C[(\Delta S_T + \Delta S_B/2)(na)^{1/2}]^{m} \cdot \frac{a^{2}/b^{2}}{}$$

(8')

Taking again account of equation (1'),

$$\frac{da}{dN} = C[(\Delta S_T + \Delta S_B/2)(na)^{1/2}]^{m} \cdot (1 - R_B \cdot a/h)^2$$

(9)

Figure 9 shows the confirmation of equation (9) in a specimen subjected to a fatigue test under combined tensile and bending loads; the relation between $a$ and $da/dN$ is given as well as that between $a$ and $N$ in integrated form. It may be noted that the experimental data agrees fairly well with equation (9).

SAFE LIFE ESTIMATION FOR PLATE THICKNESS PENETRATION OF CRACK

One of the most important objects in surface crack growth analysis is the safe life estimation against plate-thickness penetration of a crack detected on a plate surface.

Putting $a = h$ in equation (1), the value of half crack length, $b_p$, at the time of plate-thickness penetration is given as follows

$$b_p = h/(0.92 - 0.87 R_B)$$

(10)

Then, the safe life against crack penetration, $N_p$, is obtained by integration of equation (5):

$$N_p = \frac{1}{C (\Delta S_T + \Delta S_B/2)^m} \cdot \frac{\int b_p}{b_0} \cdot \frac{da}{dN}$$

(11)

where $b_0$ is the initial value of half crack length. In the case where $m \neq 2$, $N_p$ is given as follows:

$$N_p = \frac{1/h^{m/2-1} - 1/b_p^{m/2-1}}{C (\Delta S_T + \Delta S_B/2)^m \cdot (m/2 - 1)}$$

(12)

Equation (12) is valid for cracks initiated from a very short initial notch, and gives an overly safe estimation for shallow and long cracks. This equation, however, may be useful in practical application in design, where simplicity is important.

CONCLUSION

A set of empirical equations to express fatigue crack growth behaviour from a surface flaw was presented. A simplified expression for crack growth rate was proposed, other than the direct application of Paris' formula. An empirical formula was obtained to estimate the safe life against plate application in design.

REFERENCES

Table 1  Chemical Composition and Mechanical Properties

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cu</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
<th>V</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM-41</td>
<td>0.15</td>
<td>0.24</td>
<td>0.93</td>
<td>0.015</td>
<td>0.019</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HT-60</td>
<td>0.15</td>
<td>0.47</td>
<td>1.26</td>
<td>0.018</td>
<td>0.011</td>
<td>0.13</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.04</td>
</tr>
<tr>
<td>HT-80</td>
<td>0.14</td>
<td>0.25</td>
<td>0.97</td>
<td>0.014</td>
<td>0.005</td>
<td>0.27</td>
<td>tr</td>
<td>0.68</td>
<td>0.41</td>
<td>0.03</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Yield Strength (MPa)</th>
<th>Tensile Strength (MPa)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM-41</td>
<td>363.8</td>
<td>460.6</td>
</tr>
<tr>
<td>HT-60</td>
<td>519.5</td>
<td>627.2</td>
</tr>
<tr>
<td>HT-80</td>
<td>764.4</td>
<td>823.2</td>
</tr>
</tbody>
</table>

Table 2  Test Conditions

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Thickness h (mm)</th>
<th>Width w (mm)</th>
<th>Initial Notch Length 2b (mm)</th>
<th>$S_T$ (MPa)</th>
<th>$S_B$ (MPa)</th>
<th>$R_B$</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1</td>
<td>8</td>
<td>25</td>
<td>1</td>
<td>254</td>
<td>0</td>
<td>0</td>
<td>HT-60</td>
</tr>
<tr>
<td>T-2</td>
<td>8</td>
<td>25</td>
<td>1</td>
<td>254</td>
<td>0</td>
<td>0</td>
<td>HT-60</td>
</tr>
<tr>
<td>T-3</td>
<td>8</td>
<td>25</td>
<td>1</td>
<td>196</td>
<td>0</td>
<td>0</td>
<td>HT-60</td>
</tr>
<tr>
<td>T-4</td>
<td>10</td>
<td>25</td>
<td>1</td>
<td>196</td>
<td>0</td>
<td>0</td>
<td>HT-60</td>
</tr>
<tr>
<td>T-5</td>
<td>10</td>
<td>25</td>
<td>1</td>
<td>147</td>
<td>0</td>
<td>0</td>
<td>HT-60</td>
</tr>
<tr>
<td>T-6</td>
<td>10</td>
<td>25</td>
<td>1</td>
<td>196</td>
<td>0</td>
<td>0</td>
<td>HT-60</td>
</tr>
<tr>
<td>T-7</td>
<td>13</td>
<td>25</td>
<td>1</td>
<td>159.9</td>
<td>0</td>
<td>0</td>
<td>SM-41</td>
</tr>
<tr>
<td>T-8</td>
<td>13</td>
<td>25</td>
<td>20</td>
<td>169.5</td>
<td>0</td>
<td>0</td>
<td>SM-41</td>
</tr>
<tr>
<td>B-1</td>
<td>18</td>
<td>250</td>
<td>1</td>
<td>0</td>
<td>222.5</td>
<td>1</td>
<td>SM-41</td>
</tr>
<tr>
<td>B-2</td>
<td>18</td>
<td>250</td>
<td>1</td>
<td>0</td>
<td>279.3</td>
<td>1</td>
<td>HT-60</td>
</tr>
<tr>
<td>B-3</td>
<td>18</td>
<td>250</td>
<td>1</td>
<td>0</td>
<td>360.6</td>
<td>1</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-1</td>
<td>15</td>
<td>250</td>
<td>1</td>
<td>113.7</td>
<td>159.7</td>
<td>0.58</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-2</td>
<td>20</td>
<td>250</td>
<td>1</td>
<td>96</td>
<td>267.5</td>
<td>0.74</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-3</td>
<td>32</td>
<td>250</td>
<td>1</td>
<td>14.1</td>
<td>242.1</td>
<td>0.85</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-4</td>
<td>15</td>
<td>250</td>
<td>1</td>
<td>417.6</td>
<td>229.3</td>
<td>0.66</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-5</td>
<td>15</td>
<td>250</td>
<td>1</td>
<td>179.3</td>
<td>352.8</td>
<td>0.66</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-6</td>
<td>15</td>
<td>250</td>
<td>1</td>
<td>143.1</td>
<td>214.7</td>
<td>0.65</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-7</td>
<td>15</td>
<td>250</td>
<td>10</td>
<td>108.8</td>
<td>236.2</td>
<td>0.68</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-8</td>
<td>15</td>
<td>250</td>
<td>20</td>
<td>119.6</td>
<td>247.9</td>
<td>0.67</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-9</td>
<td>15</td>
<td>250</td>
<td>20</td>
<td>109.8</td>
<td>256.2</td>
<td>0.68</td>
<td>HT-60</td>
</tr>
<tr>
<td>TB-10</td>
<td>15</td>
<td>250</td>
<td>40</td>
<td>113.7</td>
<td>227.4</td>
<td>0.67</td>
<td>HT-60</td>
</tr>
</tbody>
</table>

Figure 1  Specimen geometry
Figure 2  Loading condition and stress distribution in T8-type specimen

Figure 3  Semi-elliptical surface crack in a plate

Figure 4  Relation between a/b and a/h for cracks propagating from a very short initial notch

Figure 5  Relation between a/b and a/h for cracks propagating from a longer initial notch
Figure 6  Comparison between experimental and calculated crack depth
Figure 8  Relation between $\frac{db}{dN}$ and $\frac{(\Delta S_T + \Delta S_B)}{\sqrt{N}}$.

Figure 9  Relations between $a$, $N$ and $\frac{da}{dN}$ in a specimen subjected to combined tensile and bending loads.