A MODEL FOR FATIGUE CRACK INITIATION IN POLYCRYSTALLINE SOLIDS

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1. INTRODUCTION

As is well known, fatigue is a primary cause of mechanical failure in components being elastically stressed. Numerous efforts have been made to analytically describe this important phenomenon and the resulting literature is correspondingly large. A good review of the situation, especially from the applied point of view, is the recent survey [1].

Restricting attention to probabilistic techniques, the mathematical formalisms utilized in the present work, although applied in a novel manner, are common in reliability studies in design [2] and in probabilistic structural analysis [3]. Perhaps the closest work along the lines developed here is the study carried out by Murzewski [4] who described the cumulative damage in solids under random stress. Esin and Jones [5] also presented the outline of a theory of micro-inhomogeneity of stresses and strains which result from the microstructural plastic properties of engineering materials. Certain aspects of their work are also alluded to in the present investigation.

The existence of distributions rather than deterministic values of both displacements and strains, and hence stresses, in polycrystalline materials being subjected to an externally applied load is firmly established. For example, in [6] random displacements and rotations of individual grains of aluminum embedded in an epoxy resin were observed using a combined holographic and X-ray technique, while in [7] experimental data was used to plot both probabilistic density functions and correlations of plastic deformations in quasi-isotropic polycrystalline aluminum and copper.

The most common application of probabilistic concepts in the study of fatigue is the application of specific distributions to describe the observed scatter in the number of cycles to failure. For example, Bloomer and Roylance [8] and Korbacher [9] reviewed the applicability of the log-normal, normal, Weibull [10] and extreme value [11] distributions used to describe the number of cycles to failure in polycrystalline aluminum and copper respectively. Generally they found that censored or truncated forms of these distributions give very good descriptions of the observed failure distributions especially near their "lower tails".

Recently, a probabilistic micromechanics theory, developed by Axelrad [12] and Provan and Axelrad [13] has led to the prediction of elastic microstress Gaussian probabilistic measures, $\mathcal{G}$, where $\mathcal{G}$ indicates the microstress, in realistic models of polycrystalline copper and aluminum subjected to uniaxial tension [14]. These distributions were obtained by investigations of: (i) the mechanical response of dislocations [15]; (ii) the computer simulation of the elastic behaviour of various grain boundaries in copper and aluminum [16]; and (iii) the displacement distributions presented in [6]. It was found in [14] that, for a specified dislocation density and

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Part IV - Fatigue: Mechanics

2.1 Stress Interference with Yield Strength - The Effect of Residual Stresses

With reference to Figure 1, the residual stress effect is incorporated into the model through the interference between the \( i \)th loading microstress distribution \( \sigma_{ik} \), with the yield strength distributions \( X_k \), where \( X_k \) indicates the yield strength of the \( k \)th microelement. The probability that any microelement \( \alpha \) be plastically deformed at any peak stress is the probability that \( \sigma_{ik} \) exceeds \( X_k \). Thus a new random variable, \( k^i \), is defined as:

\[
k^i = \frac{1}{X_k} \left( \frac{\sigma_{ik}}{X_k} \right)^{2/2}
\]

and has the Gaussian density, \( p_k(\cdot) \), provided \( X \) and \( e \) are both normally distributed, described by:

\[
p_k(i) = \frac{1}{2\pi e_k} \left( \frac{e - e_k}{2} \right)^{1/2} \exp \left\{ -\frac{(e - e_k)^2}{2e_k} \right\} \quad \text{for} \quad \frac{e - e_k}{2} \in X_k \quad \text{and} \quad \frac{e - e_k}{2} \in X_k.
\]

With its appropriate subscript, \( p \), indicates the probability density function of the random variable involved. Using simple transformations the fraction of microelements which become plastically deformed on the \( j \)th stress, \( Q_{ij} \), is given by:

\[
Q_{ij} = \int_{-\infty}^{\infty} p_k(i) d\alpha = \frac{1}{2\pi e_k} \left( \frac{e - e_k}{2} \right)^{1/2} \exp \left\{ -\frac{(e - e_k)^2}{2e_k} \right\} \quad \text{for} \quad \frac{e - e_k}{2} \in X_k \quad \text{and} \quad \frac{e - e_k}{2} \in X_k.
\]

The residual stress is related to the amount by which a stress exceeds the yield strength in a material. This is a manifestation of the microscopic phenomenon of dislocation generation and pile up at grain boundaries resulting in an increasing amount of residual microstress and local micro-work hardening. Although the process is macroscopically elastic, microscopically it is thermodynamically irreversible. To model this effect it is assumed that if the transition from one stress state to the next the following relations hold:

\[
\alpha_{ij}\cdot \alpha_{ij}^{-1} = \alpha_{ij}^{-1} \quad \text{for} \quad i < \alpha < X \quad \text{and} \quad j = \alpha - 1.
\]

The third interpretation is perhaps the most important from our point of view. This says that the statistics of the variables involved within the meso-domain are position independent. With this in mind, the only meso-domain we are interested in, in the present study, is the surface meso-domain which is defined as a layer of the surface material in which the statistics of the random parameters are position independent. The problem of crack propagation is not being attempted here since the meso-domain is no longer the same near a crack tip. The meso-domain contains no damaged, i.e., plastically deformed or fractured, microelements and all contribute to resisting the external load. Distributions exist for the microstress, yield strength, and ultimate strength of the microelements. If a microelement is stressed beyond its yield strength, residual stresses remain, while if it is stressed beyond its ultimate strength it is no longer effective in resisting any externally applied stress, i.e., it ceases to exist for the purposes of this analysis.
1.2 Stress Interference With Ultimate Strength - Accumulation of Damage

Again with reference to Figure 1, accumulation of damage at each load is due to the interference between $\zeta$ and the ultimate-strength distribution, $\zeta_1$, where $\zeta_i$ indicates the ultimate strength of the $i$th microelement. The probability that any microelement, $\alpha$, be fractured, thereby becoming ineffective, is the probability that $\zeta_i$ exceeds $\zeta$. Again a new random variable defined as follows may be introduced:

$$\zeta^1 = \zeta - \zeta^1.$$  \hspace{1cm} (7)

The equations already introduced, namely (2) and (3), are again applicable with $\zeta^1$ replacing $\zeta$, $\zeta$ replacing $X$, $P_1$ replacing $Q_1$, and $Z_1$ replacing $K_1$, with the understanding that $P_1$ is the fraction of the effective microelements which are permanently damaged at the $i$th stress.

Cumulative fatigue damage is modelled mathematically as follows. Consider the first loading. There are no permanently damaged microelements and hence the mean and variance of the microstresses in the surface mesodomain are:

$$\mu_0 = \mu_{0^1}; \sigma_0 = \sigma_{0^1}.$$  \hspace{1cm} (8)

At the first loading, a fraction $P_1$ of the microelements become permanently damaged. Hence, the number of damaged microelements and effective microelements, after the first load is respectively:

$$m^1 = P_1M_0; \bar{m}^1 = P_1M - m^1.$$  \hspace{1cm} (9)

Since $M_0$ is less than or equal to $M$ the average stress in the microelements before the second externally applied load is increased by an amount:

$$\bar{\mu}_2 = \mu_{2^1}/(1-P_1) + \frac{2}{\bar{m}^1}H_{\mu_1}^{\mu_{0^1}}.$$  \hspace{1cm} (10)

the first term being the Robotnov damage term of fracture mechanics. Similarly, since the number of microelements rendered ineffective during the second load is: $\bar{m}^2 = P_2\bar{m}^1$, the mean stress at the third load becomes:

$$\bar{\mu}_3 = \mu_{3^1}/(1-P_1) + \frac{2}{\bar{m}_j^2}H_{\mu_2}^{\mu_{0^2}}.$$  \hspace{1cm} (11)

Extending this to the $i^{th}$ load then:

$$\bar{\mu}_i = \frac{\mu_{i^1}}{(1-P_1)} + \frac{2}{\bar{m}_j^{i-1}}H_{\mu_i}^{\mu_{0^i}} \text{ where } \bar{m}^i = 1 - \sum_{j=1}^{i-1} \frac{1}{\bar{m}_j}.$$  \hspace{1cm} (12)

Concerning the criterion for fatigue failure, the surface mesodomain is assumed to fail when the mean stress in the microelements exceeds the mean of their ultimate strengths, i.e., if:

$$\bar{\mu}_i > \bar{\mu}.$$  \hspace{1cm} (13)
REFERENCES


Figure 1 Schematic of Interferences Between the Stress Distribution and the Distributions of Yield and Ultimate Strengths

Figure 2 Theoretical S-N Curves for Various Microstructurally Different Coppers and Aluminums

Figure 3 Random Loading Histogram and Its Resulting Distribution of Numbers of Simulated Stresses to Failure in Idealized Aluminum With a C.O.V. of 0.48
Figure 4 The Monte Carlo Simulated S-N Curves for Idealized Copper and Aluminium

Figure 5 The Monte Carlo Simulated S-N Curves for Steel