DAMAGE AND CREEP DEFORMATION IN BRITTLE ROCKS

J.F. Shao[1], K.T. Chau[2] and X.T. Feng[3]

[1]Laboratory of Mechanics of Lille, Polytech-Lille, Cite Scientifique, Villeneuve d'Ascq, FRANCE
[2] Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hong Kong, CHINA
[3]Institute of rock and soil mechanics, The Chinese Academy of Sciences, Wuhan, CHINA

ABSTRACT

A new constitutive model is proposed for the description of induced anisotropic damage in brittle rocks. The formulation of the model is based on relevant results from the micromechanics consideration. The distribution of microcracks is approximated by a second order damage tensor. The effective elastic properties of damaged material are derived from the free enthalpy function. The evolution of damage is directly related to the growth of microcracks in different space orientations. The volumetric dilatancy due to sliding crack opening is taken into account. The model is extended to the description of creep deformation in brittle rocks. The time dependent deformation is seen as a consequence of the sub-critical propagation of microcracks due to stress corrosion process. The proposed model is applied to a typical brittle rock, the Lac du Bonnet granite. A general good agreement is obtained between numerical simulations and experimental data.

1. INTRODUCTION

Anisotropic damage due to microcracks is an essential mechanism of inelastic deformation and failure in most brittle rocks. Many experimental tests have confirmed various mechanisms of nucleation and growth of microcracks. Under compressive stresses, sliding wing cracks seem to be the principal propagation mode of microcracks (Brace and Bombolakis 1963; Nemat-Nasser and Horii 1982; Wong 1982; Stef 1984; Olsson 1995). Due to roughness of crack surfaces in geomaterials, the crack sliding may induce an associated aperture which is an origin of volumetric dilatancy in these materials (Nemat-Nasser and Obata 1988). When the crack length reaches some critical values, the coalescence of microcracks occurs and localized macro-cracks appear leading to the failure of material. The kinetics of failure is controlled by confining pressure and there is a transition from brittle failure to ductile failure when the confining pressure increases (Fredrich et al. 1989; Horii and Nemat-Nasser 1985, 1986). The main consequences of induced microcracks are: non linear stress-strain relations, deterioration of elastic properties, induced anisotropy, volumetric dilatancy, irreversible strains due to residual crack opening, unilateral response due to crack closure, and hysteresis associated with frictional mechanism. These features have to be considered in the constitutive modelling. Two families of damage models have been developed for the description of induced damage: micromechanical approaches and phenomenological models (we do not give here an exhaustive list of many models reported in the literature). The main advantage of micromechanical approaches is the ability to account for physical mechanisms involved in the nucleation and growth of microcracks. For the construction of a micromechanical model, two steps are generally performed. The first step consists in the evaluation of the effective elastic properties of material weakened by microcracks. The second step is to propose a suitable damage evolution law for microcrack growth. The main features related to microcrack growth, opening and closure, friction, interaction between cracks, could be taken into account in such micromechanical models. The macroscopic behaviour of material is then obtained through a homogenization procedure. This renders these models difficult to be applied to practical applications. Phenomenological models use internal variables to represent the density and orientation of microcracks, for instance, scalar variable for isotropic damage, second and fourth rank tensor to describe anisotropic damage. The constitutive equations are generally derived from the concept of effective stress or the formulation of a thermodynamic potential. The damage evolution law is determined according to the principles of the irreversible thermodynamics. The main advantage of such models is that they provide macroscopic constitutive equations, which can be easily implemented and applied to engineering analyses. The main weakness is that some of the concepts and parameters involved in these models are not clearly related to physical mechanisms. A number of phenomenological and micromechanical models have been successfully used in the analysis of structures with metals, composites and concrete. However, most models have focused on the description of damage induced by tension-dominated stresses. Specific features of damage in brittle rocks induced by compressive stresses have not been properly taken into account. For example, many models use a tensile strain-based damage criterion. Laboratory results on brittle rocks show that such a criterion could not correctly describe the high-pressure sensitivity of such materials. Therefore, the validity of these models in the compression regime is not clearly proved. This paper presents a new phenomenological model with the emphasis on the description of anisotropic damage in brittle rocks subjected to compression-dominated stresses. The basic idea is to include relevant micromechanics features in the phenomenological formalism. Further, it is assumed that in brittle rocks, the induced damage is the essential dissipation mechanism. Plastic deformation due to dislocation can be neglected. Macroscopic irreversible strains are related to residual opening and mismatching of microcracks during loading-unloading process.

Another important feature in brittle rocks is time dependent deformation (Kranz 1979, 1980; Lajtai et al. 1987; Martin and Chandler 1994). The development of this creep deformation is an essential factor for long term safety in many structures in civil engineering and oil engineering, for instance, facilities for the storage of nuclear wastes. In the classic approaches, the creep deformation is generally described by the viscoplastic theory (Cristescu and Hunsche 1998). The viscoplastic models indeed provide a simple mathematical framework for the modeling of creep deformations, but they do not take into account physical mechanisms related to these deformations. In the case of brittle rocks, experimental data have shown that the creep deformation is essentially related to sub-critical propagation of microcrack. Physical processes related to the sub-critical propagation of microcracks have been discussed in some previous works (Atkinson 1984; Atkinson and Meredith 1987; Swanson 1984; Miura et al. 2003). In this paper, a unified approach is proposed. The modeling of creep deformation is performed as an extension of the anisotropic damage model for short terms behaviors. An evolution law is defined for the determination of the sub-critical propagation rate of microcracks. This is added to the instantaneous propagation of microcracks due to mechanical loading. The proposed model is applied to a typical brittle rock, the Lac du Bonnet granite. This class of rocks is of great interest for the feasibility study for the underground storage of nuclear waste in several countries.

2. MICROMECHANICS BACKGROUND

In this work, it is assumed that brittle materials are submitted to compression-dominated stresses. The crack density remains small and the interaction between microcracks can be neglected before the coalescence of microcracks. The initial behavior of materials is isotropic and the anisotropy is fully induced by preferential distribution of microcracks.

Consider now a representative element volume Ω (REV) of a brittle material containing sets of microcracks in different orientations. The REV is submitted to a uniform stress field on its boundary. The crack distribution in space orientation can be defined by a continuous scalar density function $\omega(\vec{n})$. The displacement jump can be decomposed into a normal component and a shear sliding component. When the crack is closed, $(\vec{n}.\mathrm{s}.\vec{n}) < 0$, the normal displacement jump vanishes.

The microcracks are assumed to be of penny-shaped. Therefore, the free enthalpy function of cracked material can be written as follows (Kachanov 1993):

$$
\begin{aligned}
W_c &= \frac{1}{2}\,\mathrm{s} : \mathrm{S}^0 : \mathrm{s} + \frac{h}{4\pi}\int_{S^{2+}} \omega(\vec{n})(1-\frac{\nu_0}{2})(\mathrm{s}.\vec{n}).<\vec{n}.\mathrm{s}.\vec{n}>^{+}\vec{n}dS \\
&+\frac{h}{4\pi}\int_{S^2} \omega(\vec{n})\{(\mathrm{s}.\mathrm{s}):(\vec{n}\otimes\vec{n}) - \mathrm{s}:(\vec{n}\otimes\vec{n}\otimes\vec{n}\otimes\vec{n}):\mathrm{s}\}dS
\end{aligned}
\tag{1}
$$

where S^0 is the initial elastic compliance tensor of undamaged material. The material coefficient is given by:

$$
h = \frac{16(1-\nu_0^2)}{3E_0(2-\nu_0)} \tag{2}
$$

In order to define a state variable of anisotropic damage at the macroscopic level, tensorial approximation methods of crack distribution are widely used. In the present work, the approximation with a second order tensor is taken as this is the simplest choice. The second order damage tensor is defined by:

$$
\boldsymbol{D} = \frac{1}{4\pi}\int_{S^2} \frac{N}{\Omega}\left(r^3 - r_0^3\right)\ (\vec{n}\otimes\vec{n})ds \tag{3}
$$

where r_0 is the average radius of initial microcracks. By using this approximation, the free enthalpy function can be expressed with the macroscopic damage tensor:

$$
W_c = \frac{1}{2}\,\mathrm{s}:\mathrm{S}^0:\mathrm{s} + a_1 tr\boldsymbol{D}(tr\,\mathrm{s})^2 + a_2 tr(\mathrm{s}.\mathrm{s}.\boldsymbol{D}) + a_3 tr\,\mathrm{s}\quad tr(\boldsymbol{D}.\mathrm{s}) + a_4 tr\boldsymbol{D}tr(\mathrm{s}.\mathrm{s}) \tag{4}
$$

where the parameters a_i are given by:

$$
a_1 = \frac{-c}{70}h\,,\ a_2 = \frac{7+2c}{7}h\,,\ a_3 = \frac{c}{7}h\,,\ a_4 = \frac{-c}{35}h \tag{5}
$$

The coefficient c is equal to $c=-\nu_0$ for the opened cracks and $c=-2$ for the closed cracks.

3. ANISOTROPIC DAMAGE MODEL

The free enthalpy function is used as the thermodynamic potential to derive the elastic damage constitutive equations. By making the standard derivative of (4), the stress-strain relations can be obtained:

$$
\mathrm{e} - \mathrm{e}^r = \frac{\partial W_c}{\partial \mathrm{s}} = \mathrm{S}(\boldsymbol{D}):\mathrm{s} \tag{6}
$$

where $\mathrm{e}^r(\boldsymbol{D})$ denotes the tensor of irreversible strains related to misfit and residual opening of microcracks. The components of the effective elastic compliance tensor are given by:

$$
\begin{aligned}
S_{ijkl}(\boldsymbol{D}) &= \frac{1+\nu_0}{2E_0}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) - \frac{\nu_0}{E_0}\delta_{ij}\delta_{kl} + 2a_1(tr\boldsymbol{D})\delta_{ij}\delta_{kl} \\
&+\frac{1}{2}a_2(\delta_{ik}D_{jl}+\delta_{il}D_{jk}+D_{ik}\delta_{jl}+D_{il}\delta_{jk}) + a_3(\delta_{ij}D_{kl}+D_{ij}\delta_{kl}) + \quad a_4(tr\boldsymbol{D})(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})
\end{aligned}
\tag{7}
$$

In the framework of thermodynamics, the damage evolution law should be determined by the formulation of a dissipation potential in the space of the conjugated forces associated with the damage tensor. In this work, a direct approach is preferred in order to facilitate the determination of model parameters. The damage evolution is directly related to the crack propagation criterion which is based on the fracture mechanics. Based on experimental data from triaxial compression tests on rocks, the crack propagation is controlled by both the mean stress (confining pressure) and deviatoric stress. For the set of cracks in the orientation $\vec{n}$, the following crack propagation criterion is proposed:

$$F(\mathrm{s},r,\vec{n}) = \sqrt{r}\left[\sigma_n\left(\frac{f_c}{f_c+\langle -\sigma_n\rangle^+}\right)^m + f(r)\ \ \tilde{q}\right] - C_{rc} \le 0 \qquad (8)$$

where σ_n is the normal stress applied to crack surfaces, which allows to take into account the pressure sensitivity of frictional materials. $\tilde{q}(\vec{n})$ is the extension stress applied to crack surfaces. This extension stress is generated by the deviatoric stress tensor and represents the driving force for the crack propagation. Note that the use of deviatoric stress components as effective stresses generating tensile cracking was originally proposed by Dragon and Mroz (1979). The bracket $\langle x\rangle^+$ denotes that $\langle x\rangle^+ = x$ for $x > 0$ and else $\langle x\rangle^+ = 0$.

In the crack propagation criterion (8), $f(r)$ is a scalar valued function controlling the kinetics of crack propagation. It plays the similar role as the hardening-softening function in plastic models. The expression of this function may be determined approximately from relevant numerical results of micromechanical models or from numerical fitting of experimental data. The general form of the function must, however, satisfy certain requirements. For small crack extents, it should decrease, reflecting the relaxation of local tensile stress as the crack grows away from the source; as the crack length becomes large enough to interact with the stress fields of other nearby cracks $f(r)$ increases or reaches an asymptotic value. The first effect causes initially stable growth and the second marks the onset of accelerated crack interaction producing damage localization and macroscopic failure. The following simple function having these basic features is here used:

$$\begin{cases} f(r) = \eta\ \ (\dfrac{r_f}{r}),\ r < r_f \\ f(r) = \eta,\ r \ge r_f \end{cases} \qquad (9)$$

where r_f is the critical crack radius for instable propagation of microcracks, and η is a parameter of model. From the propagation criterion (8), the material strength in uniaxial compression and tension can be determined:

$$f_c = \frac{C_{rc}}{\eta\sqrt{r_f}}\ ,\ \ f_t = \frac{C_{rc}}{(1+2\eta)\sqrt{r_f}}\ ,\ \ \frac{f_c}{f_t} = \frac{1+2\eta}{\eta} \qquad (10)$$

4. SUBCRITICAL CRACK PROPAGATION AND CREEP DEFORMATION

In many brittle rocks, creep deformation and time dependent failure process are observed. The modeling of creep deformation is essential in long term safety analysis of many structures, in particular constructions for nuclear waste storage. In the classic approaches, the creep deformation is described by the viscoplastic models. These models provide a simple mathematical framework but do not take into account physical mechanisms involved in rock creep. Laboratory investigations have shown that the creep deformation in brittle rocks is mainly due to sub-critical propagation of microcracks by stress corrosion. The kinetics of corrosion depends on environmental conditions. Therefore, in this model, a specific criterion is proposed for the determination of growth rate of microcracks:

$$\dot{r} = \alpha \left\langle \frac{C_r - C_{rs}}{C_{rc}} \right\rangle^{\beta} \tag{11}$$

where C_{rs} is the residual material toughness for the sub-critical crack propagation. The sub-critical propagation driving force, which is equivalent to stress intensity factor (SIF), is expressed by:

$$C_r = \sqrt{r}\left[\sigma_n \left(\frac{f_c}{f_c + \langle -\sigma_n \rangle^{+}}\right)^{m} + f(r)\tilde{q}\right] \tag{12}$$

5. NUMERICAL SIMULATIONS

The proposed model is applied to a typical brittle rock; the Lac du Bonnet granite. The parameters of the model have been determined from triaxial compression tests and creep tests (Lajtai et al. 1987). The simulations of triaxial compression tests with various confining pressures have been performed and a good agreement with experimental data has been obtained. On Figure 1, simulations of uniaxial creep tests are presented. These tests have been performed respectively on dry and wet conditions. We can see that the creep rate is higher in wet condition than in dry condition. This means that the stress corrosion process is intensified by wet condition. The proposed model correctly predicts the mechanical behaviour and creep deformation of the granite. The proposed mode seems to be able to correctly describe mechanical behavior of brittle rocks and creep deformation due to sub-critical propagation of cracks. This model is easy to be implemented in a computer code for engineering applications.

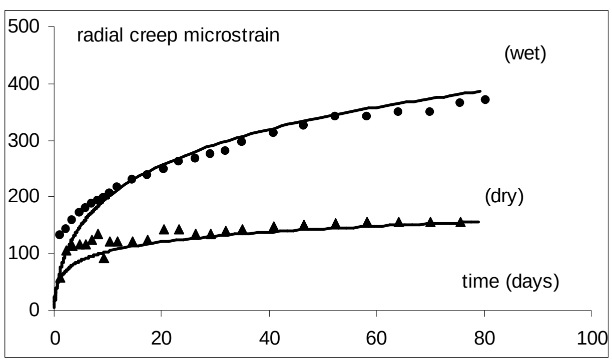

Figure 1: similations of uniaxial (continuous lines) creep tests in dry and wet conditions (data from Lajtai et al. 1987)

ACKNOWLEDGEMENTS

Acknowledgements: The work has jointly been supported by a grant from the France/Hong Kong Joint Research Programme (PROCORE N°5710QD) and a grant from the Chinese Academy of Sciences (Reference No. 2004-1-7).

RFERENCES

1. Atkinson B.K. (1984), Sub-critical crack growth in geological materials. J. Geophysical Res. 89, 4077-4114

2. Atkinson B.K., Meredith P.G. (1987). The theory of sub-critical crack growth with applications to minerals and rocks. in Fracture Mechanics of Rocks, Academic Press Inc., London, 111-166
3. Brace W.F., Bombolakis E.G. (1963) A note on brittle crack growth with applications to minerals and rocks, in Fracture Mechanics of Rocks, Academic Press, 111-166
4. Cristescu N.D., Hunsche U. (1998), Time-effects in rock mechanics, Wiley, England
5. Dragon A. & Mroz Z. (1979), A continuum model for plastic-brittle behaviour of rock and concrete, Int. J. Eng. Sci., Vol. 17, 121-137
6. Fredrich J.T., Evans B. and Wong T.F. (1989), Micromechanics of the brittle to plastic transition in Carrara marbe. J. Geophys. Res., 94, 4129-4145
7. Horii H. and Nemat-Nasser S. (1985), Compression-induced microcrack growth in brittle solids: Axial splitting and shear failure. Journal of Geophysical Research, Vol.90, No. B4, 3105-3125
8. Horii H. and Nemat-Nasser S. (1986), Brittle failure in compression: Splitting, faulting and brittle-ductile translation, Plil. Trans. R. Soc. Lond. A319, 337-374
9. Kachanov M. (1993). Elastic solids with many cracks and related problems, in Advances in Applied Mechanics, Vol. 30, Hutchinson J. and Wu T. (eds), Academic Press, N.Y., 259-445
10. Kranz R. L. (1979). Crack growth and development during creep of Barre granite. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 16, 23-35
11. Kranz R. L. (1980). The effects of confining pressure and stress difference on static fatigue of granite. Journal of Geophysical Research, 85(B4), 1854-1866
12. Lajtai E.Z., Schmidtke R.H., Bielus L.P. (1987). The effect of water on the time-dependent deformation and fracture of a granite, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 24(4), 247-255
13. Lubarda V.A., Krajcinovic D. (1993), Damage tensors and the crack density distribution. Int. J. Solids Structures, 30(20), 2859-2877
14. Martin C.D., Chandler N.A. (1994), The progressive failure of Lac du Bonnet granite, Int. J. Rock Mech. & Min. Sci., 31(4), 643-659
15. Miura K., Okui Y., Horii H. (2003), Micromechanics-based prediction of creep failure of hard rock for long-term safety of high-level radioactive waste disposal system, Mechanics of Materials, 35, 587-601
16. Nemat-Nasser S. and Horii H. (1982), Compression-induced non-planar crack extension with application to splitting, exfoliation and rock-burst. Journal of Geophysical Research, 87(B8), 6805-6821
17. Nemat-Nasser S. and Obata M. (1988), A microcrack model of dilatancy in brittle materials. ASME, J. Applied Mechanics, Vol. 55, 24-35
18. Nemat-Nasser S. and Hori M. (1993), Micromechanics: overall properties of heterogeneous materials. North-Holland ASME
19. Pensee V., Kondo D. and Dormieux L. (2002), Micromechanical analysis of anisotropic damage in brittle materials. J. Engng. Mech., ASCE, 128(8), 889-897
20. Stef P.S. (1984), Crack extension under compressive loading, Engineering Fracture Mechanics 20(3), 463-473
21. Swanson P.L. (1984), Sub-critical crack growth and other time and environment dependent behavior in crustal rocks, J. Geophys. Res. 89, 4137-4152
22. Wawersik W.R., Brace W.F. (1971). Post-failure behavior of a granite and diabase. Rock Mechanics, 3, 61-85
23. Wong T.F. (1982), ‘Micromechanics of faulting in Westerly granite’, Int. J. Rock Mech. Min. Sci. 19, 49-62