THE INFLUENCE OF BIAXIAL STRESS ON THE FATIGUE BEHAVIOR OF DEFECT-CONTAINING STEELS

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ABSTRACT

A modified linear-elastic method of analysis is proposed for the determination of fatigue strengths and fatigue lifetimes of biaxially loaded components containing defects. The method assumes that the fatigue strength and the fatigue lifetime are both controlled by fatigue crack propagation considerations, and that the fatigue cracks propagate as Mode I cracks. The modifications made to the standard method of linear analysis are as follows:

1. A modification to include consideration of elastic-plastic behavior.
2. A modification to deal with the inability of the standard approach to predict fatigue crack growth behavior in the small crack regime.
3. A modification to deal with the development of crack closure in the wake of a newly-formed crack.

A number of material constants are involved in the analysis. These include the effective fatigue crack growth threshold, ΔK_{effth}; the crack opening level for a macroscopic crack, K_{opmax}; and k, a constant which determines the rate of crack closure development with crack advance.

Experiments were carried out under combined cyclic loading conditions employing specimens of two steels which differed in their mechanical properties and which contained simulated surface defects of various sizes to check on the accuracy of the predictions. It was found that both the predicted fatigue strengths as well as the predicted fatigue lives agreed well with the experimental results.

1 INTRODUCTION

The design engineer often has to make estimates of the fatigue strengths and fatigue lives of components which will be subjected to biaxial loading based only upon limited data obtained under uniaxial conditions. Therefore there is some uncertainty concerning the reliability of such estimates, particularly when defects may be present in the components. To improve reliability it may necessary to go through the costly experimental process of obtaining the fatigue properties under simulated service conditions. It would therefore be helpful if a reliable means were available to be able to predict fatigue behavior of components containing small defects under various biaxial stress conditions based upon a minimum of uniaxial data, such as the yield strength and the fatigue limit. The analysis presented in the present paper represents a step in this direction, and includes experimental results to serve as a check on the analysis. The analysis assumes that the fatigue lifetime is essentially spent in crack propagation; that is, any crack initiation period is small with respect to the total fatigue life and can be neglected. In addition it is assumed that fatigue cracks propagate as Mode I cracks. Experiments (Endo [1]) have shown fatigue cracks emanating from simulated defects under biaxial stress conditions of loading propagate in a direction approximately perpendicular to the maximum principal stress, lending support to the above assumption. To carry out the analysis certain material constants must be known or estimated. These include the effective stress intensity factor range at threshold, ΔK_{effth}, a material constant k, which determines the rate at which crack closure develops, and K_{opmax}, the crack opening level of a macroscopic crack. For a given alloy group such as steels, estimates of the magnitude of these constants can be made based upon prior work with similar alloys. For example, ΔK_{effth} will always be close to 3.0 MPa$\sqrt{\text{m}}$ for steels.

2 ANALYSIS

The basic equation used in the analysis is (McEvily, Eifler and Macherauch [2]):

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A(\Delta K_{\text{eff}} - \Delta K_{\text{effth}})^2, \tag{1}$$

which can also be written in the form

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A\left[\Delta K_{\text{I, biaxial}} - (\Delta K_{\text{op}} + \Delta K_{\text{effth}})\right]^2, \tag{2}$$

where A is a material constant of the order of 4×10^{-10} (MPa$\sqrt{\text{m}}$)$^{-2}$ for steels, and

$$\Delta K_{\text{I, biaxial}} = \left[\left(1 - \frac{\sigma_2}{\sigma_1}\right)\sqrt{2\pi r_{\text{e}} F_{\text{biaxial}}} + Y\sqrt{\pi a F_{\text{biaxial}}}\right]\Delta\sigma_1, \tag{3}$$

where σ_1 and σ_2 are principal stresses, r_{e} is a material constant of the order of 1 μm (see below) which is introduced to compensate for the fact that a standard linear elastic analysis is not able to deal with short cracks, and Y reflects the particular crack geometry involved. For a semi-circular surface crack Y has a value of 0.73. F_{biaxial} is introduced to deal with elastic-plastic behavior, and is defined as

$$F_{\text{biaxial}} = \frac{1}{2}\left(\sec\frac{\pi\sigma_1}{2\sigma_{\text{Y, biaxial}}} + 1\right) = \frac{1}{2}\left[\sec\frac{\pi\sigma_1}{2\sigma_{\text{Y}}}\sqrt{1 - \frac{\sigma_2}{\sigma_1} + \left(\frac{\sigma_2}{\sigma_1}\right)^2} + 1\right]. \tag{4}$$

where $\sigma_{\text{Y,biaxial}}$ and σ_{Y} are the yield stresses under biaxial and uniaxial loading, respectively. The following equation is used to characterize the development of crack closure in the wake of a newly formed crack

$$\Delta K_{\text{op}} = (1 - e^{-k\lambda})(K_{\text{opmax}} - K_{\text{min}}), \tag{5}$$

where λ is the length of the actual fatigue crack, and K_{opmax} is the crack opening level for a macroscopic crack, and $K_{\text{min}} = -\Delta K_{\text{I, biaxial}}/2$ for R = -1 loading.

The value of r_{e} is calculated from the following equation:

$$\Delta K_{\text{I, uniaxial}} = \left(\sqrt{2\pi r_{\text{e}} F} + Y\sqrt{\pi r_{\text{e}} F}\right)(2\sigma_{\text{w0}}) = \Delta K_{\text{effth}}, \tag{6}$$

where σ_{w0} is the uniaxial fatigue strength defined to be the minimum stress amplitude at which a crack can propagate to failure, and

$$F = \frac{1}{2}\left(\sec\frac{\pi\sigma_{w0}}{2\sigma_Y} + 1\right). \tag{7}$$

3 EXPERIMENTAL PROCEDURES

The materials investigated were an annealed 0.37 % carbon steel (JIS S35C) and a quenched and tempered Cr-Mo steel (JIS SCM435). The chemical compositions are listed in [1]. The mechanical properties measured after heat-treatments are shown in Table 1.

The smooth specimens had a uniform cross section of either 8.5 or 10 mm in diameter, and 19 mm in length. After heat treatment a 30 μm thickness of surface layer was removed by electro-polishing, a drilled hole or a crack as shown in Fig. 1 was then introduced into the surface to simulate a defect. Both the major axes of in-line holes and the crack faces were normal to the maximum principal stress. It was experimentally confirmed that the shape of a crack was approximately semi-circular in torsion as well as in tension-compression. A single cylindrical hole is referred to as a 1-hole defect and a defect connected with two or three adjacent holes is referred

Table 1: Mechanical properties.

	Lower yield point (MPa)	Tensile strength, (MPa)	Reduction in area (%)	Vickers hardness *HV*
S35C	328	563	47.5	160
SCM435	851	947	65.7	324

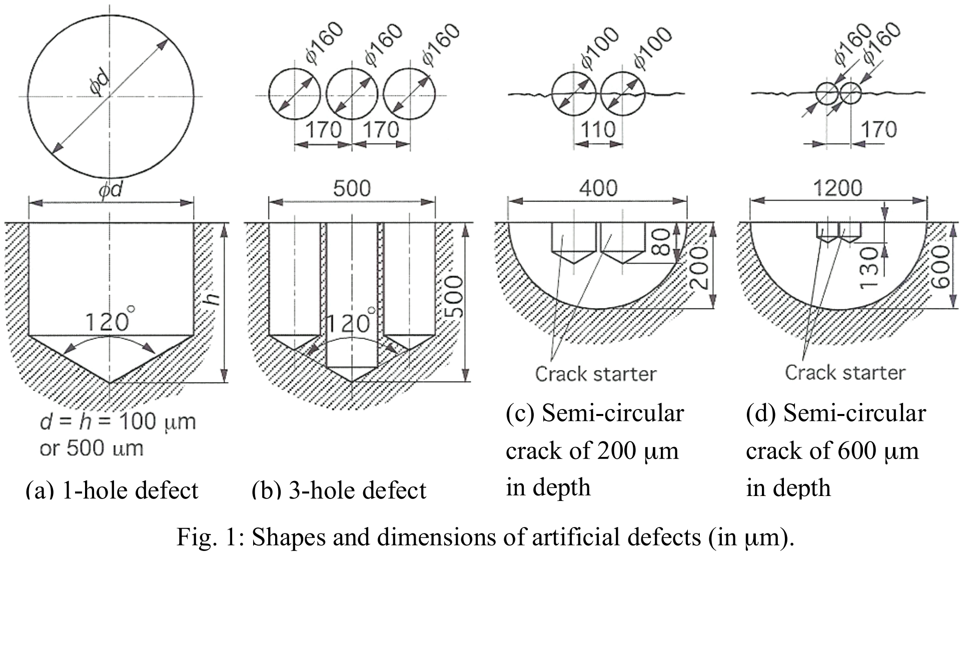

Fig. 1: Shapes and dimensions of artificial defects (in μm).

to as a 2-hole or 3-hole defect. In the case of SCM435, fatigue cracks were initiated at the 2-hole defects, a crack starter, and grown to total lengths of either 400 μm or 1200 μm. After pre-cracking, the specimens were annealed at 600 °C in vacuum to eliminate any residual stresses. The $\sqrt{area}$ parameter model (Murakami [3]) was used to convert the initial defect size of 1-hole defect or a 3-hole defect into an equivalent the crack depth, a_0, for a semi-circular crack by use of the relation $a_0 = \sqrt{2/\pi}\sqrt{area}$.

Tension-compression tests were carried out using a servo-hydraulic uniaxial testing machine with an operating frequency of 50 Hz. The torsional and combined load tests were performed using a servo-hydraulic axial/torsional testing machine operating at 30-45 Hz. All tests were conducted under in-phase fully reversed (R = -1) loading using a sinusoidal waveform. The ratio of the amplitude of shear stress to amplitude of normal stress, τ/σ, was chosen to be either 0, 1 or ∞, which corresponds to principal stress ratios, σ_2/σ_1, of 0, -0.382 and –1, respectively.

4 RESULTS AND DISCUSSION

4.1 Fatigue strengths

Figures 3 and 4 show a comparison of the predicted and experimental fatigue limits of specimens containing defects. The endurance limit for a smooth specimen under biaxial loading was related to the experimentally known endurance limit under axial loading by the following equation:

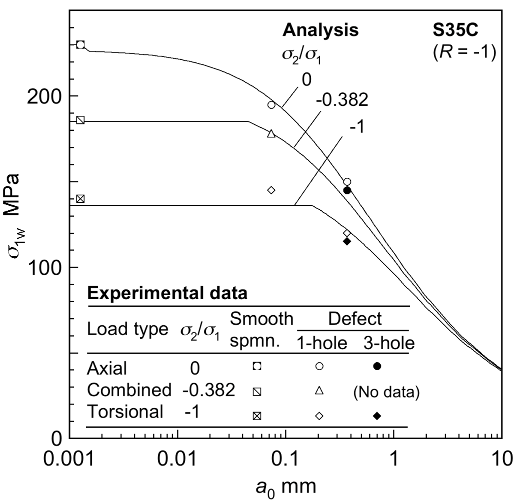

Fig. 2: Relationship between principal stress amplitude at fatigue limit and initial defect size (S35C).

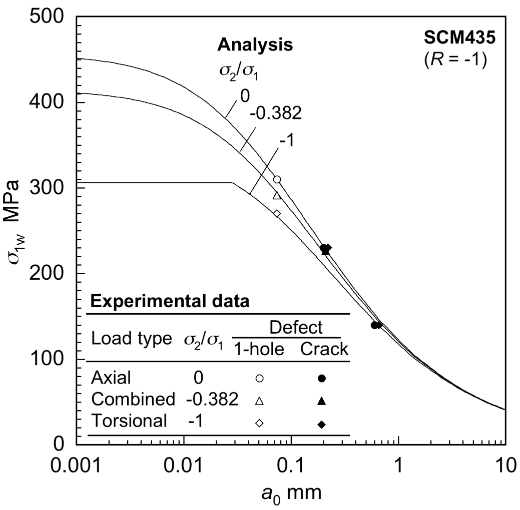

Fig. 3: Relationship between principal stress amplitude at fatigue limit and initial defect size (SCM435).

$$\left[\left(1-\frac{\sigma_2}{\sigma_1}\right)\sqrt{2\pi r_e F_{\text{biaxial}}} + Y\sqrt{\pi r_e F_{\text{biaxial}}}\right](2\sigma_1) = \left(\sqrt{2\pi r_e F} + Y\sqrt{\pi r_e F}\right)(2\sigma_{w0}). \tag{8}$$

Since the value of σ_{w0} is known from experiments, and r_e and F can be obtained from eqns (6) and (7), respectively, the right hand side of eqn (8) can be evaluated. For a given ratio of σ_2 to σ_1 the left hand side of eqn (8) can then also be evaluated.

The predicted fatigue strengths for specimens containing defects were calculated by setting da/dN in eqn (2) equal to zero; that is,

$$\Delta K_{\text{I, biaxial}} - \left(\Delta K_{\text{op}} + \Delta K_{\text{effth}}\right) = 0. \tag{9}$$

Equation (9) can be written in expanded form as

$$\begin{aligned} &\Delta K_{\text{I, biaxial}} - (1-e^{-k\lambda})(K_{\text{opmax}} - \frac{1}{2}\Delta K_{\text{I, biaxial}}) - \Delta K_{\text{effth}} = 0, \text{ or} \\ &\Delta K_{\text{I, biaxial}} - \frac{2\left[\Delta K_{\text{effth}} + (1-e^{-k\lambda})K_{\text{opmax}}\right]}{1+e^{-k\lambda}} = 0. \end{aligned} \tag{10}$$

$\Delta K_{\text{I,biaxial}}$ was calculated from eqn (3), and the material constants used in the calculations are given in Table 2. In carrying out the calculations, it was found that the calculated fatigue strength for specimens containing small defects sometimes exceeded the fatigue strength of a smooth specimen. In such cases the fatigue strength of a smooth specimen was substituted for the calculated value. This resulted in the horizontal lines shown in Figs. 3 and 4.

4.2 Lifetime predictions

A relationship between the initial defect length, a_0, and number of cycles to failure, N_f, as a function of stress amplitude was obtained by integrating eqn (1) between the limits a_0 and the final crack length, a_f, with the value of a_f taken to be 5 mm. Figure 4 shows the *S-N* curves for SCN435 pre-cracked specimens, in which a comparison of calculated results and experimental data is made. The value of A of $4\times10^{-10}(\text{MPa}\sqrt{\text{m}})^{-2}$ was used to provide a best fit. It is noted that in the case of SCM435, the influence of load types on the *S-N* curves was relatively small as compared to S35C at the same a_0 value. This is because in the case of SCN435 the applied stresses are smaller compared to the yield stress (σ_Y = 851 MPa) than in the case of S35C (σ_Y = 328 MPa) and therefore the elastic-plastic effects is much smaller.

Table 2: Material constants used in the analysis.

	σ_Y MPa	σ_{w0} MPa	A $(\text{MPa}\sqrt{\text{m}})^{-2}$	k m^{-1}	K_{opmax} $\text{MPa}\sqrt{\text{m}}$	ΔK_{effth} $\text{MPa}\sqrt{\text{m}}$	r_e μm
S35C	328	230	4×10^{-10}	20000	3.0	2.5	1.3
SCM435	851	518*	4×10^{-10}	60000	3.0	2.5	0.3

* The fatigue strength of smooth specimens of SCM435 was estimated by use of the relation $\sigma_{w0} = 1.6HV$.

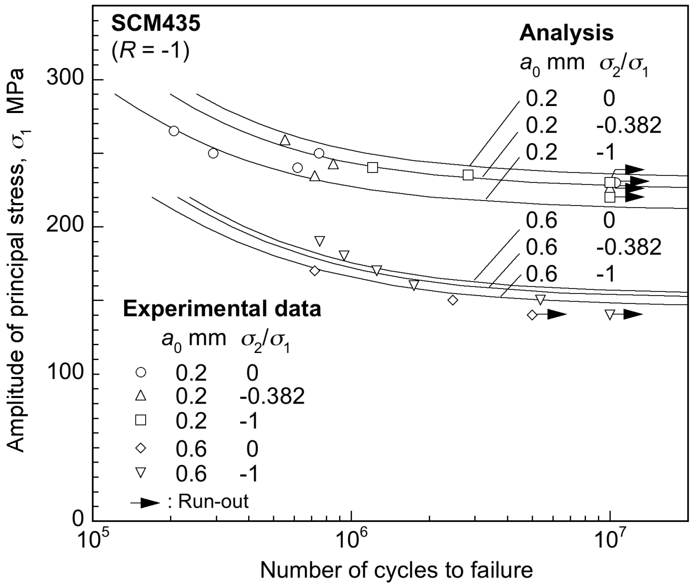

Fig. 4: *S-N* curves of pre-cracked specimens (SCM435).

5 CONCLUDING REMARKS

1. A method of analysis for determining the fatigue strength of specimens containing defects under biaxial loading conditions has been proposed. Good agreement between predictions based upon this method and experimental results has been obtained.
2. The method has also been used to predict the fatigue lives of specimens containing defects under biaxial loading. Again good agreement between experiment and predictions was obtained.

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