

# NUMERICAL THREE-DIMENSIONAL CRACK GROWTH SIMULATION FOR COMPONENTS WITH MULTIPLE CRACKS

M. Fulland<sup>1</sup> and H.A.Richard<sup>2</sup>

<sup>1</sup>Westfälisches Umwelt Zentrum, Paderborn, Germany

<sup>2</sup>Institute of Applied Mechanics, University of Paderborn, Germany

## ABSTRACT

The following paper deals with the issue of the automatic simulation of the propagation of multiple cracks in a three-dimensional structure. Besides the general problem of performing a full 3D-FE-crack growth simulation, the handling of multiple cracks yields some special problems, which arise from the fact, that the crack growth rates of the cracks might be very different from each other. Consequently the at first sight proximate idea of propagating all cracks in each simulation step is not a reasonable approach for the problem, since extremely small increments for some cracks would result in enormous numerical instabilities. In the paper the approach followed by the crack simulation program ADAPCRACK3D is discussed in detail. It realises crack growth for multiple cracks by “real” propagation of only one crack within one simulation step and “virtual” increments for all other crack fronts. The applicability of this approach is proven by a simulation example of two corner cracks in a bending beam.

## 1 INTRODUCTION

In the application of most three-dimensional crack growth simulation tools only the “one-crack-in-one-structure” problem is under consideration. However it is quite likely especially in huge components, that more than only one crack exists within the structure. Even if the different cracks do not directly interact with each other, the lifetime of a structure and the development of the cracks cannot reasonably be estimated with only investigating the different cracks “one-by-one”. For this reason it is essential to carry out three-dimensional crack growth simulations, which are able to handle the existence of multiple cracks in a structure.

## 2 FUNCTIONALITY OF ADAPCRACK3D

The program system ADAPCRACK3D, which is able to perform fully automatic crack growth simulations in three-dimensional structures consists of three independent modules (Figure 1). The first module NETADATP3D comprises all mesh manipulation work, that has to be done throughout a simulation process. This work covers the insertion of a new crack (first simulation step) respectively the insertion of the crack propagation (all following steps) into the FE-mesh of the structure as well as the necessary mesh adaptation and especially mesh improvement, which is of major importance in the simulation sequence in order to keep the FE meshes in sufficiently good quality. The FE solution itself is operated by the commercial FE-program ABAQUS<sup>TM</sup>. On the basis of this FE-solution finally the fracture mechanical evaluation is carried out. In the module NETCRACK3D the stress intensity factors and thus the crack growth direction and rate as well as the necessary number of loading cycles for the crack propagation are determined. As major result of NETCRACK3D new crack front coordinates are obtained, which are sent back to the first module in order to continue the simulation. A detailed description of the functionality of the program can be gathered from [1] and [2].

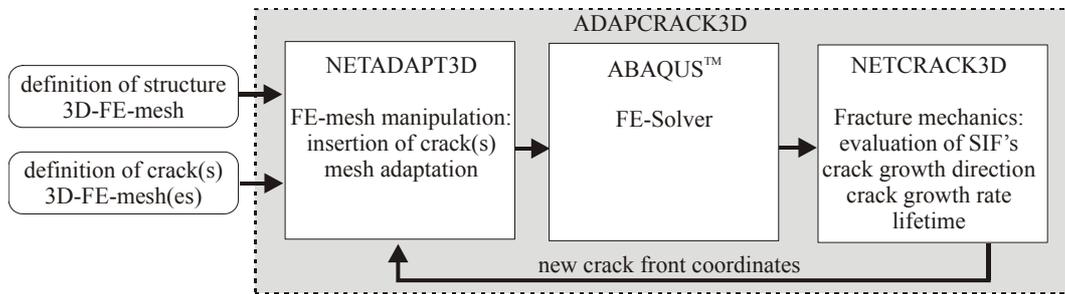


Figure 1: Simplified functionality scheme of ADAPCRACK3D

**3 CRACK GROWTH SIMULATION AND LIFETIME ESTIMATION FOR MULTIPLE CRACKS**  
 The fracture mechanical evaluation in ADAPCRACK3D is based on the concept of linear elastic fracture mechanics. So the determination of stress intensity factors  $K_I$ ,  $K_{II}$  and  $K_{III}$  for all three fracture modes has to be accomplished first for any node of any of the crack fronts of the given problem. This is performed by the use of the MVCCI-method [3], that yields an excellent accuracy and numerical stability.

### 3.1 Simulation sequence for one single crack

Before discussing the handling of multiple cracks within a structure it is reasonable to explain the general simulation sequence for just one single crack: As soon as the stress intensities are calculated, the crack growth direction at any node of the crack front is given by application of the  $\sigma_1$ '-criterion by Schoellmann et al. [4]. The crack growth increment along the crack front is determined by the following procedure: Besides the propagation direction the  $\sigma_1$ '-criterion defines a cyclic comparative stress intensity factor  $\Delta K_v$  as a function of the stress intensity factors and the stress ratio  $R$  for any node of the crack front. The crack growth rate  $da/dN=f(\Delta K_v, R, \Delta K_{th}, \dots)$  for those nodes is calculated either by the law of Erdogan/Ratwani [5] or the Forman/Mettu-equation [6].

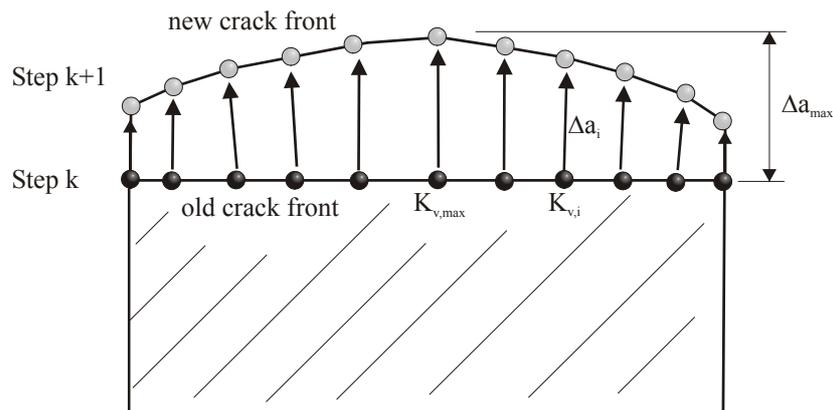


Figure 2: Determination of crack growth increment for any node of a crack front

The particular node, which is subjected to the biggest  $\Delta K_v$ , is propagated by the user-defined maximum crack growth increment per step  $\Delta a_{\max}$  (Figure 2). By  $\Delta a_{\max}$  over  $da/dN(\Delta K_{\max})$  a (coarse) estimation of the number of loading cycles  $N_{\text{estim}}$ , which are necessary for such increment, is given. So the smaller increments  $\Delta a_i$  for any other node of the crack front now result from the product of  $N_{\text{estim}}$  and the local crack growth rate  $da/dN(\Delta K_i)$ . The combination of the local crack growth increment and the local crack propagation direction finally yields new crack front coordinates, which are reapplied in ADAPCRACK3D's first module in order to continue the simulation with step k+1 (comp. Figure 1).

After the completion of the subsequent step k+1 a reliable lifetime calculation for the incrementally growing crack according to Figure 2 can be established. The number of loading cycles for the crack increment from step k to k+1 are given by the following integration:

$$N_{k \rightarrow k+1} = \int_0^{\Delta a_{\max}} \frac{da}{f(\Delta K_v, R)} \quad (1)$$

In eqn. (1) the cyclic stress intensity and thus the crack growth rate are known for the start and end point of the integration. For sufficiently small increment  $\Delta a_{\max}$  the crack growth rate can be assumed to vary linearly over the integration interval, which enables to perform a numerical evaluation of eqn (1).

### 3.2 Simulation sequence for multiple cracks

If more than one crack is to be investigated within a structure the crack propagation procedure described above has to be adapted. Since the different cracks might have extremely different crack growth rates, the at first sight proximate idea of propagating all cracks at once in each simulation step would cause some cracks to obtain only very small increments, which would cause enormous numerical instabilities in the simulation. So in ADAPCRACK3D a modified approach for this problem is applied: The general idea is only to propagate one crack per simulation step in *reality*, while storing *virtual* number of loading cycles for all non propagated cracks. For reasons of simplicity the simulation process for multiple cracks is explained in the following for three cracks within a structure, but is identically applicable for an arbitrary number of cracks.

For multiple cracks in ADAPCRACK3D three different “types” of cycle numbers have to be taken into consideration:

- $N_{\text{estim}}$  defines the estimated number of loading cycles for each crack to accumulate a crack growth of  $\Delta a_{\max}$ . It is calculated according to the explanation in section 3.1.
- $N_{\text{loc}}$  defines the “local” number of loading cycles for each crack, that has been applied to it in order to propagate it up to its current size.
- $N_{\text{glob}}$  is the global number of loading cycles, which has been applied to the whole structure.  $N_{\text{glob}}$  always is the maximum of the different  $N_{\text{loc}}$ .

In the first simulation step the crack, which holds the smallest  $N_{\text{estim}}$  is propagated. In the example of Figure 3 this is true for the first crack. At this point of the simulation all  $N_{\text{loc}}$  and  $N_{\text{glob}}$  are still zero. In the second simulation step now the true number of loading cycles for crack 1 can be calculated by eqn. (1), which is stored as  $N_{\text{loc}}$  for crack 1 as well as  $N_{\text{glob}}$  (7500 in the example). Usually this “true” number of loading cycles is smaller than the estimated one in the step before, since in most cases the stress intensity and thus the crack growth rate increases with a growing crack. Once again the number  $N_{\text{estim}}$  for the subsequent crack growth step is estimated for all cracks (7000-10000-12000). In the example it is assumed, that the cracks do not directly interact with each other, which explains, that the number  $N_{\text{estim}}$  does not change for the cracks 2 and 3. However,  $N_{\text{estim}}$  is recalculated for any crack in each simulation step. The next crack to be propagated is determined by the following formula:

$$P = N_{\text{estim}} - (N_{\text{glob}} - N_{\text{loc}}). \quad (2)$$

The minimum of  $P$  denotes the crack, that will grow in the particular step. In this equation the difference  $(N_{\text{glob}} - N_{\text{loc}})$  can be interpreted as “virtual” number of loading cycles, which concede the non-propagated cracks of a certain step an advance for the next one. In the example of Figure 3 the minimum of  $P$  is found for crack 2 by  $P = 10000 - (7500 - 0) = 2500$ . All following steps are performed in exactly the same way as step 2. So the procedure described above is able to handle a three-dimensional crack growth simulation with an arbitrary number of cracks under inclusion of a reasonable lifetime calculation while avoiding numerical instabilities resulting from possible extremely different crack growth rates for the different cracks.

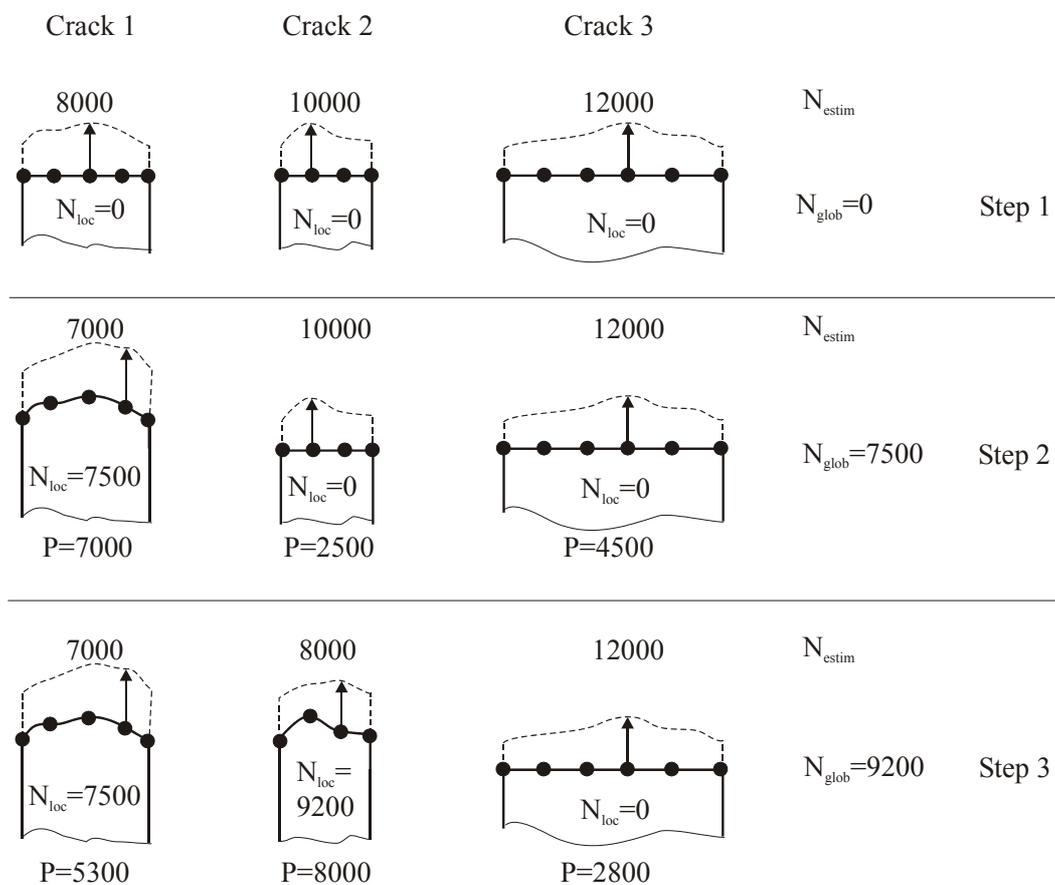


Figure 3: Crack propagation for multi-cracks on basis of virtual local number of loading cycles

#### 4 APPLICATION EXAMPLE

The simulation example by ADAPCRACK3D is performed for a bending beam with two quarter circular corner cracks of radius  $r = 8$  mm as specified in Figure 4. The simulated material is an aluminium alloy AlZnMgCu1,5 T652 with a fracture toughness  $K_{Ic} = 973$  N/mm<sup>3/2</sup>. The outcome of this simulation can be gathered from Figure 5. It can be seen, that the first crack with a distance of

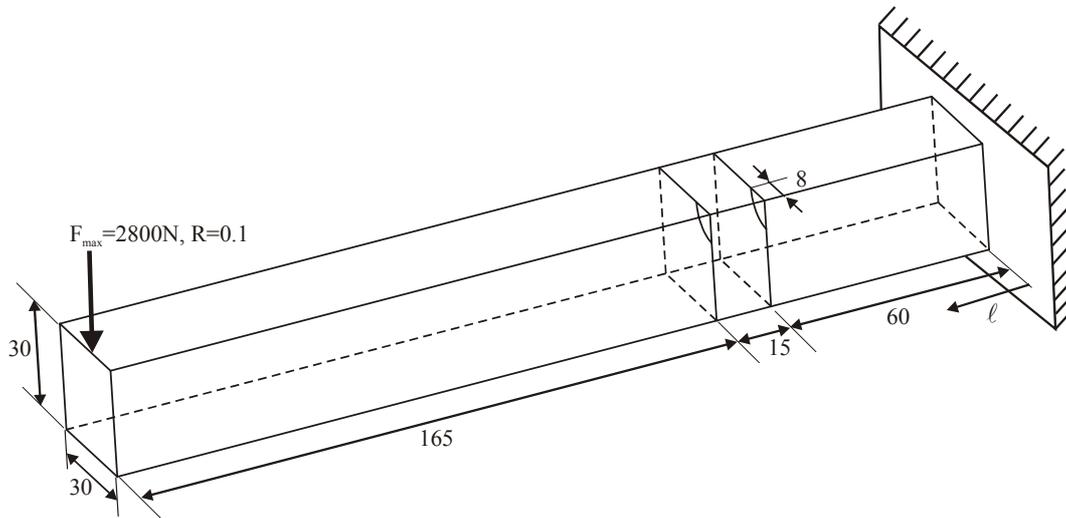


Figure 4: Bending beam with two quarter circular corner cracks

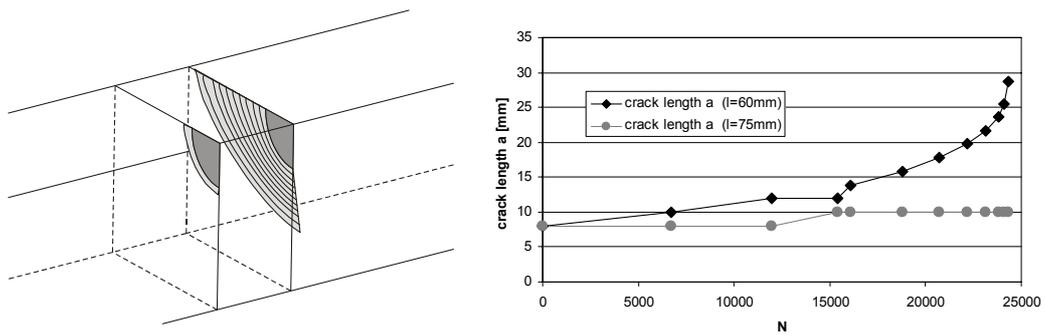


Figure 5: Development of the cracks in multiple-crack-simulation

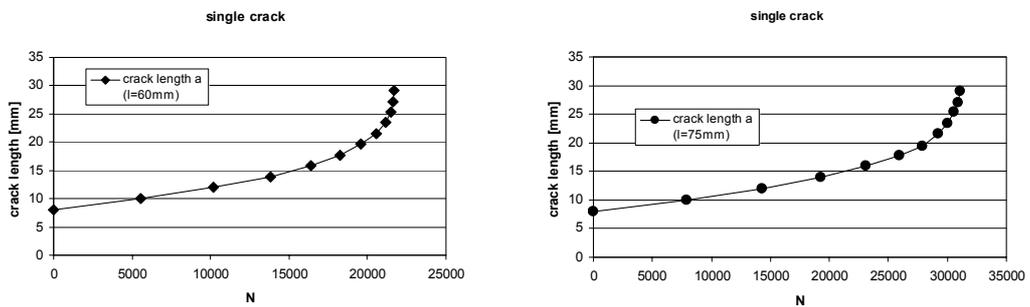


Figure 6: Development of the cracks in two independent „one-crack-simulations“

$l=60\text{mm}$  from the clamping is the dominating one, which is growing very fast. The second crack only obtains one crack growth increment during the simulation. At a crack length of  $28,5\text{mm}$  the first crack becomes unstable after a number of ca. 24000 loading cycles. If the result of such a “multiple-crack-simulation” is compared to the results of two independent “one-crack-simulations” (Figure 6) some notable differences become apparent. Despite the fact, that the critical crack size in each simulation is approximately the same, the number of calculated loading cycles for crack 1 ( $l=60\text{mm}$ ) differs about 10%. This means that the “one-crack-simulation” is too conservative in this case. If the development of crack 2 ( $l=75\text{mm}$ ) is under consideration, the differences are evidently bigger. Even if in the “one-crack-simulations” the number of applied loading cycles is restricted to  $N=22000$  (at this point crack 1 fails), the resulting crack length for crack 2 ( $a\approx 15\text{mm}$ ) does not match the real crack length obtained by the multi-crack-simulation ( $a\approx 10\text{mm}$ ) at all. So the resulting crack shape for the non critical cracks obviously have no significance in a simple “one-crack-simulation”.

## 5 CONCLUSIONS

The crack simulation program ADAPCRACK3D is capable of handling the task of performing a full three-dimensional crack growth simulation for a structure with multiple cracks. The presented algorithm avoids the numerical instabilities possibly resulting from very different crack growth rates by a propagation sequence, which is based on virtual loading cycles for the different cracks, if the crack growth increment is too small. The simulation example proves the applicability of this algorithm to industrial problems. It becomes apparent, that the outcome of a “multiple-crack-simulation” may be notably different from the results of several “single-crack-simulations” even if the cracks do not interact directly.

## 6 REFERENCES

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