PROBABILISTIC FATIGUE ANALYSIS FOR STRUCTURAL HEALTH MONITORING

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ABSTRACT

A probabilistic fatigue damage analysis procedure which is coupled to a structural health monitoring system is discussed. The conceptual monitoring system includes a pair of ultrasonic sensors - a narrowband SAW generator and a harmonically matched SAW receiver. The sensors provide information of the current level of damage by monitoring the second harmonic in the SAW signal as a function of loading and number of cycles. This information is then transferred to the probabilistic fatigue damage analysis procedure for probabilistic forecasting of the damage evolution of the component. Here we report preliminary results obtained by applying this methodology to experimental data.

1 INTRODUCTION

The usefulness of the damage-tolerance philosophy for life predictions of components is critically dependent on the ability of the NDE technique to accurately and reliably monitor damage. In materials such as high strength steels undergoing fatigue, critical damage in the form of a crack of detectable but very small length, often occurs late in the lifetime of a component. When a detectable crack has developed out of microscopic damage processes, it grows to an unacceptable length in a time that is short as compared to the total lifetime of the component. Therefore in order to apply the damage-tolerant philosophy in such cases, it is essential to develop a monitoring system to monitor pre-crack damage. The present paper foresees a structural health monitoring system to monitor the pre-crack damage, whose sensor output is used in a fatigue damage evolution model to predict the remaining life of the component. Probabilistic damage deveopment is evaluated using Monte Carlo integration with Importance Sampling.

2 STRUCTURAL HEALTH MONITORING SYSTEM

In the approach of this paper, the structural health monitoring system is represented by a pair of harmonically matched ultrasonic transducers which measure the acoustic nonlinearity in the component. The acoustic nonlinearity A_2/A_1, defined as the ratio of the second harmonic amplitude A_2 to the fundamental amplitude A_1, quantifies the extent to which an ultrasonic wave is distorted as it propagates through a material that has been subjected to fatigue loading (see e.g. Morris et. al. [1]). Ogi et. al [2] have observed that the acoustic nonlinearity increases nearly monotonically, and shows a distinct peak at the point of macro-crack initiation. This phenomenon can be attributed to the changes in the microstructure which is a direct consequence of the accumulated fatigue damage.

3 DAMAGE MODEL

This section presents a damage model whose evolution represents the evolution of the non-linearity up to the point of macrocrack initiation. The state of damage in a specimen at a particular cycle during fatigue is represented by a scalar damage function $D(N)$. The magnitude $D = 0$ corresponds to no damage, and $D = 1$ corresponds to the appearance of the first macrocrack. The following phenomenological model, which is a modification of the model proposed in Bolotin [3](pg. 98), is assumed to represent the evolution of the damage

$$\frac{dD}{dN} = \begin{cases} \dfrac{1}{N_c}\left(\dfrac{\sigma_{max} - r_c(\bar{\sigma})}{r_c(\bar{\sigma})}\right)^m \dfrac{1}{(1-D)^{f(\sigma_{max},\bar{\sigma})}} & \text{if} \quad \sigma_{max} > r_c(\bar{\sigma}) \\ 0 & \text{if} \quad \sigma_{max} < r_c(\bar{\sigma}) \end{cases} \tag{1}$$

Here, N_c is a normalizing constant, σ_{max} is the maximum stress in a cycle, $r_c(\bar{\sigma})$ is the endurance limit when the mean stress in a cycle is $\bar{\sigma}$, $m > 0$ is a material parameter and $f(\sigma_{max}, \bar{\sigma}) > 0$ is a function of the stress. The dependence of the exponent of the damage variable on the stress ensures that the model leads to nonlinear accumulation of damage (see Lemaitre and Chaboche [4], pg. 420). In the present paper it is assumed that $f(\sigma_{max}, \bar{\sigma}) = n\,\sigma_{max}/r_c(\bar{\sigma})$. Both m and n are estimated using nonlinear regression. It is also assumed that $r_c(\bar{\sigma})$ follows the Goodman relation (see Goodman [5]), i.e.

$$r_c(\bar{\sigma}) = r_c(0)\left(1 - \frac{\bar{\sigma}}{\sigma_{ult}}\right),$$

where σ_{ult} is the ultimate tensile strength of the material. For cyclic loading where σ_{max} and $\bar{\sigma}$ are constant with σ_{max} always greater than $r_c(\bar{\sigma})$, Eq. (1) can be solved to obtain

$$D(N) = 1 - \left[(1-D_0)^{f(\sigma_{max},\bar{\sigma})+1} - \frac{N}{N_c}\left(\frac{\sigma_{max} - r_c(\bar{\sigma})}{r_c(\bar{\sigma})}\right)^m (f(\sigma_{max},\bar{\sigma})+1)\right]^{\frac{1}{f(\sigma_{max},\bar{\sigma})+1}}. \tag{2}$$

Here D_0 is the initial damage present in the specimen. To find the number of cycles needed for macrocrack initiation, $D = 1$ is substituted in Eq. (2) to obtain

$$N_{ini} = \frac{N_c}{f(\sigma_{max},\bar{\sigma})+1}(1-D_0)^{f(\sigma_{max},\bar{\sigma})+1}\left(\frac{r_c(\bar{\sigma})}{\sigma_{max} - r_c(\bar{\sigma})}\right)^m. \tag{3}$$

4 PROBABILITY OF MACROCRACK INITIATION

The procedure for calculation of the probability of macrocrack initiation is described in this section. Depending on the problem under consideration, the quantities appearing in Eq. (3) are suitably randomized. Let $\mathbf{X} = [X_1 \quad X_2 \dots X_k]^T$ denote the random quantities (for example, for a problem with known constant stress cycles, the quantities $r_c(\bar{\sigma})$, D_0, m and n can be considered random with known probability distribution and $\mathbf{X} = [X_1 \;\; X_2 \;\; X_3 \;\; X_4]^T = [r_c(\bar{\sigma}) \;\; m \;\; n \;\; D_0]^T$). Let $f_{\mathbf{X}}(\mathbf{x})$ denote the joint probability distribution of $\mathbf{X}$. To determine the probability of macrocrack initiation P_{ma}, i.e. the probability that the number of cycles to macrocrack initiation, N_{ini}, will be less than a specified number of cycles N_s, one first defines a limit state surface given by

$$g = N_{ini} - N_s.$$

To account for the inspection process, let $Ninsp$ denote the cycle number at which an inspection is carried out and let $Dinsp$ denote the damage at that cycle. If no macrocrack is observed at $Ninsp$ then it follows that $Dinsp < 1$. To account for the inherent scatter in the damage measurements, the following inequality

$$Dinsp < Dactual < 1 \tag{4}$$

is assumed, where $Dactual$ is the actual damage in the specimen at $Ninsp$. Note that this is a conservative approach and it is possible that $Dactual < Dinsp$. Also note that since it is assumed that the model represents the evolution of the damage exactly, one has $D(Ninsp) = Dactual$, where $D(Ninsp)$ is the damage predicted by the model (see Eq. (2)) at $N = Ninsp$. Therefore the inequality in Eq. (4) can be replaced by

$$Dinsp < D(Ninsp) < 1. \tag{5}$$

Let E denote the event $Dinsp < D(Ninsp) < 1$. Then the probability of macrocrack initiation P_{ma}, taking into account the inspection at $Ninsp$, is given by

$$P_{ma} \equiv Pr(N_{ini} < N_s | E) = \frac{Pr((N_{ini} < N_s) \cap E)}{Pr(E)}. \tag{6}$$

To calculate this probability, the two probabilities occurring on the right hand side of Eq. (6) are evaluated separately. To do this, it is first necessary to represent the event E in the space of random variables. This is achieved by defining a function $h(\mathbf{x})$, such that

$$h(\mathbf{x}) = \left[(1 - D_0)^{n+1} - \frac{Ninsp}{N_c}\left(\frac{\sigma_{max} - r_c(\bar{\sigma})}{r_c(\bar{\sigma})}\right)^m (f(\sigma_{max}, \bar{\sigma}) + 1)\right]^{\frac{1}{f(\sigma_{max},\bar{\sigma})+1}}.$$

From Eq. (2) it follows that Eq. (5) is equivalent to $Dinsp < 1 - h(\mathbf{x}) < 1$. This statement is equivalent to

$$0 < h(\mathbf{x}) < 1 - Dinsp$$

which represents the event E in the space of random variables. Eq. (2) shows that at $N = Ninsp$, the surface $h(\mathbf{x}) = 0$ corresponds to $D(Ninsp) = 1$, and the surface $h(\mathbf{x}) = 1 - Dinsp$ corresponds to $D(Ninsp) = Dinsp$. The probability of the event E is now given by

$$Pr(E) = \int_{0<h(\mathbf{x})<1-Dinsp} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{7}$$

and

$$Pr((N_{ini} < N_s) \cap E) = \int_{(g(\mathbf{x})<0)\cap(0<h(\mathbf{x})<1-Dinsp)} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}. \tag{8}$$

To evaluate the integrals appearing in Eqs. (7) and (8), the random variables are first mapped via a Rosenblatt transformation (see Rosenblatt [6]) into a standard Gaussian space where the random variables denoted by $\mathbf{U} = [U_1 \;\; U_2 \ldots U_k]^T$ are independent, normally distributed and have zero mean and unit standard deviation. The modified Hasofer-Lind, Rackwitz-Fiessler (HL-RF) algorithm described in Kiureghian and Liu [7] is used to obtain the point closest to the origin on the surface $g(\mathbf{u}) = 0$ which is denoted by $\mathbf{u}^*$. In the modified HL-RF algorithm, one adjusts the step size during each iteration to obtain a sufficient decrease in the merit function which is based on the first order optimality conditions. Monte Carlo integration with importance sampling, with the sampling density centered at $\mathbf{u}^*$ (see Fujita and Rackwitz [8]) is then used to calculate the integrals in Eqs. (7) and (8).

5 SAMPLE PROBLEM

The procedure described in the paper is applied to the data, viz. acoustic nonlinearity as a function of number of cycles, (e. g. see Ogi, et.al [2] for details) and the probability of macrocrack initiation is calculated. The yield strength of the material is 333 MPa specimen and it is subjected to a maximum bending stress of 280 MPa. The endurance limit of the material at zero mean stress, $r_c(0)$, is assumed to be 180 MPa. Table (1) shows the acoustic nonlinearity measured as a function of number of cycles. It is observed that the evolution of

Table 1: Measured Values of Acoustic Nonlinearity during Successive Inspections

j	$Ninsp_j$	$(A_2/A_1)_j \times 10^{-3}$
0	0	0.90
1	11200	0.80
2	22400	0.90
3	26880	1.50
4	30800	2.00
5	33040	2.50
6	34000	3.10

Table 2: 'Measured' Values of Damage during Successive Inspections

j	$Ninsp_j$	$Dinsp_j$
0	0	0
1	11200	0.2462
2	22400	0.2769
3	26880	0.4615
4	30800	0.6154
5	33040	0.7692
6	34000	0.9539

the acoustic nonlinearity is not strictly monotonic during the initial stages of fatigue. The damage is obtained by normalizing the nonlinearity measurements by the expected maximum value of the nonlinearity. This assumes that there exist a linear mapping between the acoustic nonlinearity and the accumulated damage. For the given problem the maximum value is assumed to be 3.25×10^{-3}. It is also assumed that the specimen is initially undamaged, i.e. $Dinsp_0 = 0$. The 'measured' damage which is calculated from the corresponding nonlinearity measurements is tabulated in Table (2). Using the values of damage given in Table (2), the parameter m and n are calculated using nonlinear regression (e.g. see Draper and Smith [9]). The values are calculated starting from the third inspection. It is assumed that the damage values from $0 \ldots j$ inspections are available to calculate m and n at the j inspection. The probability of macrocrack initiation is then calculated by assuming that σ_{max}, $\bar{\sigma}$, N_c and D_0 are fixed quantities while $r_c(0)$, m and n are independent random quantities each having a lognormal distribution The mean and standard deviation of m and n is obtained from nonlinear regression. The following values are used for the fixed quantities: $\sigma_{max} = 280$ MPa, $\bar{\sigma} = 0$ MPa, $N_c = 10000$, $r_c(0) = 180$ MPa and $D_0 = 0$. The random variable $r_c(0)$ is assumed have a mean of 180 MPa with a standard deviation of 5.4 MPa. The probability of macrocrack initiation is calculated as described in Section after each inspection for different N_s and is given in Table (3).

Note that for the fatigue problem described, the first macrocrack is observed at approximately 34160 cycles (see Ogi et. al. [2]). As seen from Table (3), the formation of the macrocrack is predicted quite well in spite of using a simple damage model and making simple assumptions regarding the parameters involved in the model. The results presented in the present paper also compare favourably with the results presented in Kulkarni et. al. [10] in which a different modification of Bolotin's damage accumulation model (see Bolotin [3]) is used.

Table 3: Calculation of P_{ma}

Cycles	P_{ma}			
N_s	3rd Insp (Ninsp = 26880)	4th Insp (Ninsp = 30800)	5th Insp (Ninsp = 33040)	6th Insp (Ninsp = 34000)
30000	0.04353	0.00000	0.00000	0.00000
31000	0.06201	0.05629	0.00000	0.00000
32000	0.08248	0.32226	0.00000	0.00000
33000	0.10558	0.54982	0.00000	0.00000
34000	0.13155	0.72169	0.84881	0.00000
35000	0.15705	0.83928	0.99456	1.00000
40000	0.32432	0.99690	1.00000	1.00000

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REFERENCES

[1] Moris, W. L., Buck, O. and Inman, R. V. 1979. “Acoustic harmonic generation due to fatigue in high-strength aluminum”, *J. of App. Phy.*, **50(11)**, 6737-6741.

[2] Ogi, H., Hirao, M. and Aoki, S. 2001. “Noncontact monitoring of surface-wave nonlinearity for predicting the remaining life of fatigued steels”, *J. of App. Phy.*, **90(1)**, 438-442.

[3] Bolotin, V. 1999. *Mechanics of Fatigue*, CRC Press, London.

[4] Lemaitre, J. and Chaboche, J. L. 1985. *Mechanics of Solid Materials*, Cambridge University Press, Cambridge.

[5] Goodman, J. 1899. *Mechanics Applied to Engineering*, Longmans Green, London.

[6] Rosenblatt, M. 1952. “Remarks on a multivariate transformation”, *Annals of Math. Stat.*, **23**, 470-472.

[7] Kiureghian, A. and Liu, P. January 1989. “Finite-Element reliability methods for geometrically nonlinear stochastic structures”, Report No. UCB/SEMM-89/05, Dept. of Civil Engineering, University of California, Berkeley.

[8] Fujita, M. and Rackwitz, R. 1988. “ Updating first- and second-order reliability estimates by importance sampling”, *Structural Safety/Earthquake Eng.*, **5**, 53-59.

[9] Draper, N. R. and Smith, H. 1998. *Applied Regression Analysis* (3rd Edn.), John Wiley & Sons, New York.

[10] Kulkarni, S. S., Sun, L., Moran, B., Krishnaswamy, S. and Achenbach, J. D. 2004. “ A conceptual HUMS of fatigue damage in rotorcraft components”, *2nd European Conference on Structural Health Monitoring System*, Munich, Germany.