EFFECTIVE PROPERTIES OF SOLIDS WITH STRESS CORROSION CRACKS

M. TOUZET, I. AUBERT, F. LE POULAIN, M. PUIGGALI
Laboratoire de mécanique physique UMR CNRS 5469
Université Bordeaux 1, 351 Cours de la liberation, 33405 Talence cédex, France.

ABSTRACT

One way for assessing the overall degradation of cracked solids consists in determining the effective elastic properties of the structure. Some approaches allow determining the effective elastic properties from the behaviour at the microscopic scale. These approaches require the definition of parameters characterizing the orientation, the number, the size and the shape of cracks. These parameters depend on the method chosen to determine the effective elastic properties. We apply two methods (incremental scheme and non interaction method) for the modelling of stress corrosion growing cracks. We show that it is necessary to define a crack (called active crack) which is representative of the growing cracks because we observe during experiments that propagation rate of cracks may vary depending on the number and the length of cracks. Then, we determine by the effective media theory, the changes of mechanical behaviour of a damaged stainless steel. We discuss the choice of the representative crack : mean crack, active crack and we compare experimental results and simulations.

1 INTRODUCTION

Surface cracks are the most common flaws observed in engineering structural components. Such cracks occur frequently in colonies and can be due to fatigue, fatigue corrosion (FC) or stress corrosion (SC). Among the existing ways to predict the safety of the structure, experimental approaches have been favoured in SCC and FC studies. Nevertheless, another way for studying the overall degradation of solids with cracks consists to use approximate schemes integrating or not crack interactions. The non-interacting approximation was developed by Kachanov [1]. The author considers that each crack is embedded into a matrix loaded by the same externally applied stress and that other cracks have not any influence on the studied crack. The author considers that both interaction effects (shielding and amplification) may be neglected due to the competition between them.

The homogenization methods take into account crack interactions. The methods of effective matrix where each crack is surrounded by a matrix of reduced stiffness (self-consistent scheme [2, 3], differential scheme [4, 5]) and the methods of effective field where a representative crack is placed in the undamaged matrix and loaded with an effective stress field (method of Mori-Tanaka [6]). In the methods of effective matrix, crack interactions are predicted to always reduce the effective stiffness, compared to that calculated with the interactions neglected [2]. Thus, the stiffening impact of the shielding interactions is ignored. In the methods of effective field, the shielding mode of interactions is not disregarded and in the simplest version (Mori-Tanaka), the results coincide with the method of non-interacting cracks when cracks are randomly located. Carvalho et al. [7] compared all these methods by confronting them with experimental results. They observed that the non-interacting approximation gave the best results as the competing effects of the interactions counterbalance each other.

Nevertheless, when cracks propagate, interactions can change because relative positions of cracks are continuously modified. The objective of this paper is to discuss and to compare, using two methods (incremental scheme and non interaction method), the modelling of the mechanical degradation of a material with stress corrosion surface cracks. This study will particularly focused

on the choice of the suitable parameters used in each modelling i.e. crack density parameter in the non-interaction method and representative crack in the incremental scheme.
Experimental results are presented in order to discuss the validity of the methods chosen to determine the effective elastic properties.

2 EFFECTIVE LINEAR ELASTIC PROPERTIES

A two-dimensional elastic isotropic matrix perforated by N parallel elliptical holes represents the material with surface stress corrosion multiple cracks.

Non-interacting cracks approximation
We first consider the approximation of non-interacting cracks developed by Kachanov [2] to determine the effective elastic properties. In this approach, cracks have not necessary the same length . A closed form of the effective longitudinal Young modulus [8] is given by :

$$E^{*} = \frac{E}{(1 + p + 2\rho)} \qquad (1)$$

where $p = \frac{\pi}{A}\sum_i a_i b_i$ is the porosity and $\rho = \frac{\pi}{A}\sum_i a_i^2$ is the crack density parameter of the material, A the total reference area (including cracks), a_i and b_i are respectively the half-length and the half-opening of the crack i.

Interacting cracks approximation
We use the incremental self-consistent scheme (ISCS) adapted by Broohm [9] to determine the effective elastic properties. We consider a two-phase material constituted by an isotropic material with N identical inclusions distributed uniformly and we treat the problem of an ellipsoidal inclusion embedded in an equivalent medium. The inclusion represents the crack and the equivalent medium represents the homogenized material. We can obtain, by the superposition of N identical elementary problems, the effective moduli as follows :

$$C_{eff} = C^{M} - fC^{M}\left[I + T^{II}_{eff} C_{eff}\right] \qquad (2)$$

where C_{eff} is the elastic modulus tensor of the homogenized material, C^{M} is the elastic modulus tensor of the undamaged material, f is the volume fraction of cracks and T^{II}_{eff} is the interaction tensor depending in particular on the inclusion geometry. In order to avoid the divergence of the self-consistent model [9], the incremental scheme applied to the self-consistent model was proposed by Vieville et al [10] and consists in a progressive construction of the material. The determination of the elastic effective properties is realized including at each step a finite increment of volume fraction.

In this approach (interacting crack approximation), all cracks have the same length and the same shape. We will discuss the choice of a « representative crack » that we will use to model this problem.
In the case of non-interacting crack approximation, the parameters used (porosity p and crack density parameter ρ) can't be chosen because they are dictated by the modelling.
We discuss, with experimental results proposed below, the choice of parameters representing correctly the modification of elastic effective properties.

3 EXPERIMENTAL RESULTS

To obtain quantitative characterization of surface cracks evolution, slow strain rate tests were performed using 304L stainless steel in boiling 44% $MgCl_2$ solution at 153°C.
In-situ observations through a corrosion cell with a CCD camera were performed in order to characterize quantitatively changes in each surface crack length and crack location.

Crack nucleation is a continuous process as it is shown in figure 1 for slow strain rate tests performed at $\dot{\varepsilon} = 6.10^{-6} s^{-1}$. The total number of cracks observed on the surface is presented and compared to the "active" crack density that represents the density of growing cracks between two instants of the surface examination.
In the first part of the test, initiation of cracks is very progressive and the active crack density is similar to the total crack density.
The rate of nucleation decreases until the end of the test. This result is classically observed. Nevertheless, the evolution of the active crack density is more remarkable. This feature displays the same trend but clearly shows that active crack density diminishes before the total crack number.

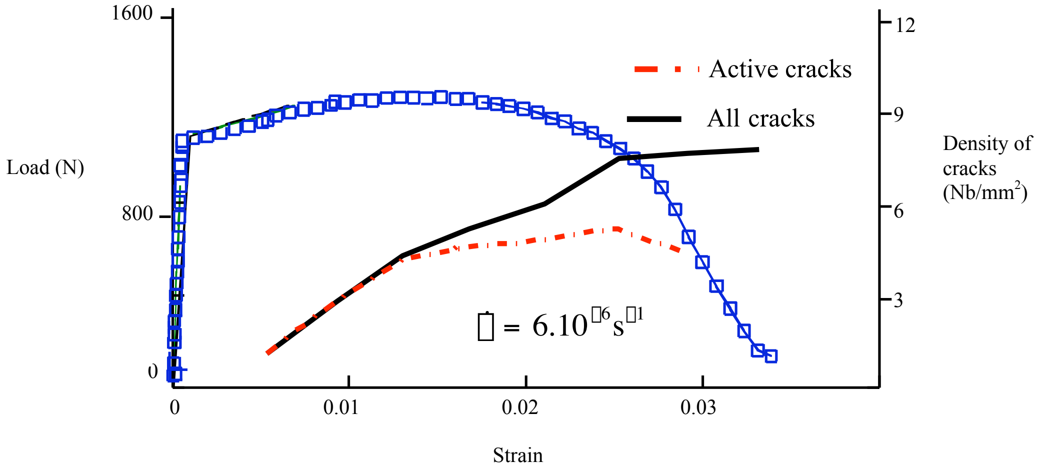

Figure 1 : Changes of crack density and active crack density during slow strain rate tests.

This observation is used to calculate, during the test, the evolution of the "active crack length" obtained by the ratio between the cumulated length of the active cracks and the number of active cracks. Figure 2 represents a comparison between lengths of the "active crack" and the mean crack length (ratio between the cumulated length of all cracks and the total density of cracks). In addition, we present the evolution of the "representative crack" calculated using the expression of the crack density parameter as follows:

$$a_{rep} = \sqrt{\frac{A}{\pi N}}$$

This figure 2 shows that, at the beginning of the test, the difference between the active and the mean crack lengths is not important. The representative crack length, which minimizes the impact of the short cracks, is longer than the others at the beginning of the test. Then, the longer one is the active crack which takes into account the stop or the increase of the propagation rate of some cracks during the test. These experimental results allow determining, the elastic effective properties of the cracked material, using the various crack evolutions previously proposed.

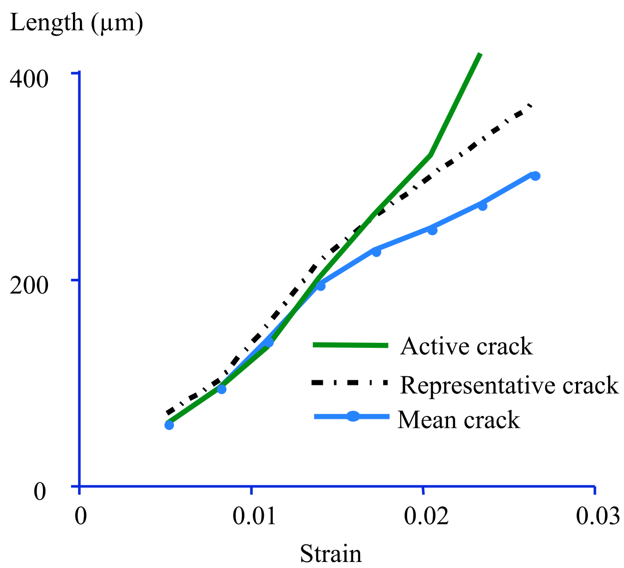

Figure 2 : Evolution of the length of the mean crack, the active crack and the representative crack during a slow strain rate test performed at $\dot{\varepsilon} = 6.10^{-6} s^{-1}$.

The effective Young's modulus was calculated with the non-interacting approximation (eq (1)) and compared to the simulation obtained with the incremental self-consistent scheme (eq(2)). In this case, the simulation was realized with the active and the mean cracks. Results are presented in figure 3.

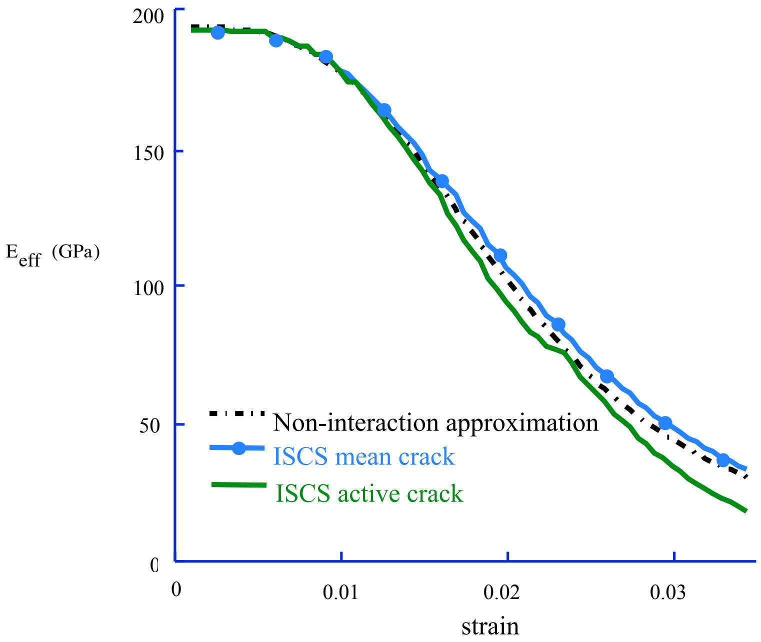

Figure 3 : Evolution of the Young's modulus using non-interaction approximation and incremental self-consistent scheme (ISCS).

4 CONCLUSION

The results of the simulation presented in figure 3 show that all the proposed models give very close results. The incremental self-consistent scheme applied to stress corrosion cracks is a good alternative to avoid divergence problems [9] or underestimation of effective properties [7] due to cracks when the self-consistent method is used.
In the non-interaction approach, the crack density parameter allows to minimize the effect of short cracks.
In the incremental self-consistent scheme, the results obviously depend on the crack size. The active crack takes into account the stop of the increase of the propagation rate of some cracks. This parameter is in better accordance with individual behaviour of cracks than the mean crack. It allows a good description of the contribution of the cracks to the changes of effective properties.

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