THE EXTENDED FINITE ELEMENT METHOD FOR STATIC AND DYNAMIC CRACK PROPAGATION

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ABSTRACT

Methods for treating cracks that are arbitrarily aligned with the mesh are described. This allows for arbitrary placement of cracks and the treatment of growing cracks without remeshing. The methodology employs a local partition of unity in which basis functions that incorporate the discontinuity and in some cases a neartip enrichment by asymptotic neartip fields. The extension of these methods to dynamic crack propagation and nonlinear shell problems is described here. Examples are given which demonstrate the robustness and versatility of the method.

1 INTRODUCTION

Methods for describing arbitrary discontinuities are a key ingredient to successful modeling of both stationary and growing cracks. In addition, it is desirable to be able to easily introduce asymptotic neartip fields for elastic fracture mechanics problems. The extended finite element method (XFEM) developed in Belytschko et al [1] and Moes et al[2] has been quite successful at meeting these needs without requiring remeshing. Some recent improvements in the method can be found in Stazi et al [3] where the method was extended to higher order elements. Sukumar et al [4], Moes et al [5] and Gravouil et al [6] extended the method to three dimensional problems by using level sets to track the evolution of the crack. An extensive survey can be found in Karihaloo and Xiao [7].

The method is based on a local partition of unity through which discontinuous functions, namely the Heaviside step function, and the basis for the neartip field can be introduced. However, for purposes of computational efficiency, the partition of unity is always local: only the elements that are cut by the crack or in the vicinity of the crack tip are enriched. This adds some complications for enrichments other than the step function, as discussed by Chessa et al [8], where remedies forthese difficulties are also proposed.

In this paper, we describe new methods for treating crack growth in shells undergoing large displacements. Here the decomposition into continuous and discontinuous fields is applied somewhat differently in order to meet the requirement of director inextensibility. We also consider methods for dynamic crack propagation. For these applications, a new method for treating the elements containing the crack tip had to be developed. The method requires the crack to be straight inside each element, but the direction of the crack can change from element to element, so the path is almost arbitrary. The dynamic XFEM method has only been developed for three node triangular elements so far.

2 METHODOLOGY

The treatment of the two classes of problems of interest here, nonlinear shells and dynamic crack propagation, necessitated the development of extensions of the standard XFEM formulation of the displacement field. In the shell problem, a Mindlin-Reissner theory based on directors is employed. Therefore, it is necessary to construct the displacement field so that the directors remain inextensible. This was accomplished in [9] by using the following director field

The displacement field for dynamic crack propagation is constructed by superimposing a triangular element with one edge coincident with the crack tip on the element that contains the crack tip. Within the superimposed element, the standard XFEM formulation for a discontinuity is used, but the classical terms are not. This avoids the difficulty of shifting from one type of enrichment to another that would be the case for a branch function based on the harmonic that is used in [1]. Details can be found in [10].

In both cases the discrete equations are obtained by the principle of virtual work. The static equations are solved by a damped Newton method, whereas the dynamic equations are solved by explicit time integration. Cohesive models are used for the crack once the advance of the crack is indicated. It is important to note that in any formulation that corresponds to a continuum formulation, a cohesive law is not sufficient to describe the propagation of a crack. As indicated in [10], a law for indication the direction and speed of the crack must be given to obtain closure of the governing equations. In this sense, the interelemenet crack models in which cracking is only permitted between elements achieve closure through the discretization, which is undoubtedly one of the reasons for the strong dependence between solutions and mesh that is found in those methods.

$$\mathbf{u} = \sum_{K=1}^{4} N_K\left(\xi\right)\mathbf{u}_K + \sum_{K=1}^{4} \frac{h\xi_3}{2} N_K\left(\;\right) \underbrace{\left[\mathbf{e}_{1K}\frac{\sin\theta_K}{\theta_K}\theta_{K2} - \mathbf{e}_{2K}\frac{\sin\theta_K}{\theta_K}\theta_{K1} + \mathbf{e}_{3K}\left(\cos\theta_K - 1\right)\right]}_{\Delta\mathbf{t}_K} \tag{1}$$

where $\mathbf{u}_K$ are nodal displacements, $\Delta\mathbf{t}_K$ are nodal directors and θ_{Ki} are the nodal rotations and $N_K(\;)$ are the shape functions. The nodal rotation norm, θ_K is defined by $\theta_K = \sqrt{\theta_{K1}^2 + \theta_{K2}^2}$.

The term $\Delta\mathbf{t}_K$ in (1) identifies the components of the director increment, $\Delta\mathbf{t}_K = \mathbf{t}_K(\;_K) - \mathbf{e}_{3K}$ with $\;_K = \left\{\theta_{K1}, \theta_{K2}\right\}$. Parameterization (1) can be directly derived from the director shell theory by imposing thickness inextensibility, or $\delta\mathbf{t}_K \cdot \mathbf{t}_K = 0$ (unitary directors will maintain that property under deformation).

Next we let

$$u_{Ki} = u_{Ki}^{std} + H_K\left(f\left(\;\right)\right)u_{Ki}^{*} \tag{2}$$
$$\theta_{Ki} = \theta_{Ki}^{std} + H_K\left(f\left(\;\right)\right)u_{Ki}^{*}$$

where $f(\;) = 0$ defined the crack. The above is equivalent to standard XFEM for the membrane displacements, but is different for the rotations; the difference is crucial, see [9].

3 GROWTH OF CRACK IN NONLINEAR SHELL

This example is based on the tests carried out by Keesecker et al. [11]. The example consists of of a crack growing in a thin closed cylindrical shell subject to spatially uniform internal pressure. The cylinder is reinforced with two tear straps (see the above reference) whose purpose is to induce ‘flapping'', which consists of crack turning near these tear straps. If the purpose is fulfilled, axially propagating cracks are arrested. The problem here is analyzed as a equilibrium problem; dynamic effects are not considered.

The geometry is presented in figure 1, along with the material constants. Other details of the model can be found in [9]. Under the effect of internal pressure, the initial crack propagates longitudinally until the tear straps restrain the hoop strain and induce crack turning (or bifurcation, as experimentally verified in the above reference).

A sequence of four deformed meshes is presented in Figure 2. It is interesting to note the bulging effect that occurs during the self-similar stage of the analysis. After the crack path turns near the strap, the bulging gradually disappears. The self-similar growth zone is imposed, but the crack path curves near the straps and the subsequent path are fully captured by the present model, even though we make the simplification of a sandwich shell model.

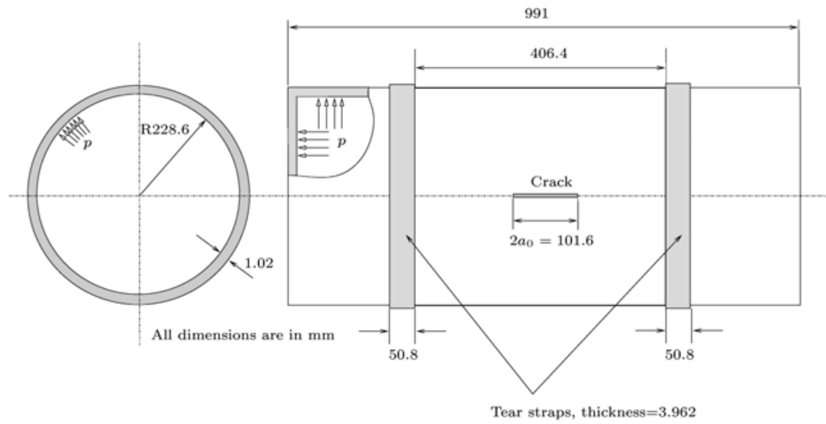


Figure 1: Geometry, loading and relevant material properties for the pressurized shell problem, see [11]

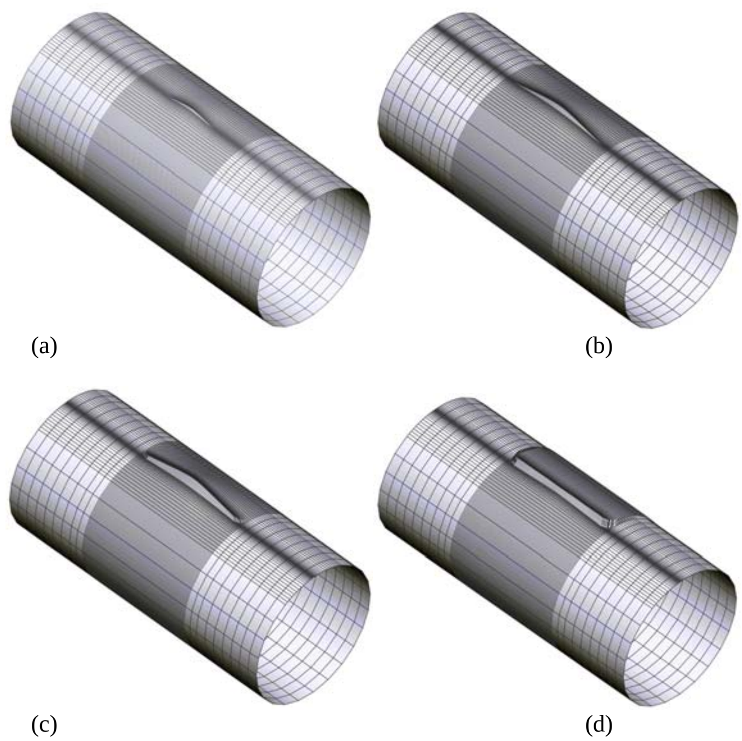


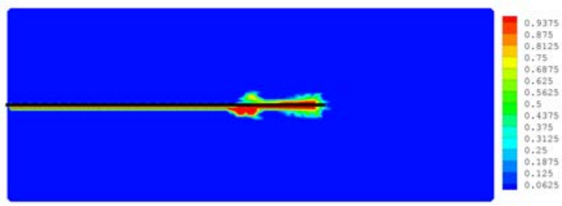
Figure 2: Pressurized shell: four deformed meshes (not magnified): (a) self-similar stage a = 86.36 mm; (b) self-similar stage a = 177.382 mm; (c) curved stage $a_{long} \approx 198.12$ mm; $a_{circumf} \approx 39.5$ mm, (d) final deformed mesh

4 DYNAMIC CRACK PROPAGATION

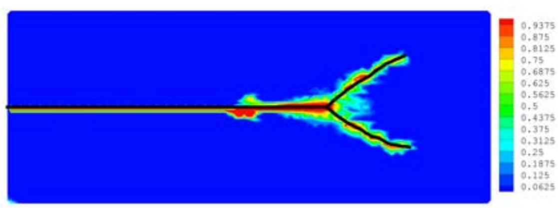
We consider a problem of a plate with an initial notch. A traction is applied instantaneously to the top and bottom edges of the plate. Experimental results for problems of this type have been obtained by Ravichander and Knowles. The plate was modeled by triangular elements. A Lemaitre isotropic damage model was used for the material, and the crack was propagated when the partial differential equation lost hyperbolicity in the vicinity of the crack tip, see [10] for details. This is a very simple damage model, but it proved quite effective in reproducing the salient features of the experiments, though it should be added that a tensile stress criterion gave similar results.

The crack evolution is shown in Figure 3. It can be seen that the crack propagates to the right, and from Figure 4 it can be seen that the speed of the crack increases in this stage. When the crack speed reaces a certain point, the hyprbolicity criterion indicates crack propagation in directions that deviate from the straight line, and we allow the crack to branch as shown. The two branches then continue to propagate to the right.

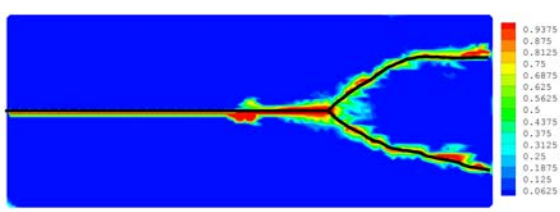
The crack speed is shown in Figure 4. It can be seen that the crackspeed always remains below the Rayleigh wavespeed, although prior to branching there is a rapid acceleration of the speed of the cractip. The crackspeed is here compared to another method now under development that employs an elementwise modeling of the crack. The two methods agree quite well, although the elementwise crack model accelerates earlier and branches earlier. Then in the later part of the simulation, the two methods agree quite well.



(a) t = 30.03 μs



(b) t = 46.00 μs



(c) t = 60.27 μs

Figure 3: Evolution of crack for a plate loaded at the top and bottom edges showing crack branching

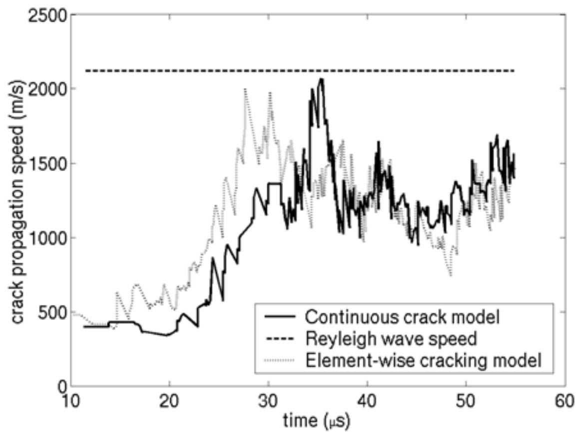


Figure 4: Crack speed for crack branching problem

5 CONCLUSIONS

The applicability of XFEM to nonlinear shell problems and dynamic crack propagation problems has been demonstrated. These have entailed the development of new ways of constructing the discontinuous displacement field, but they fit nicely in the general framework. We have not discussed the update of the level set that describes the crack. In Reference [6] this was accomplished by integrating the hyperbolic conservation equations that govern the level set. This unfortunately is quite burdensome and furthermore requires special procedures the freeze the surface of the crack surface that has already developed. In Ventura et al [12], techniques are developed that enable the level set to be updated by geometric equations and automatically freeze the preexisting crack surface. The work on dynamics in [10] also brought out to us that XFEM can be viewed as a form of superposition, and that therefore one can contruct a crack model simply by superimposing elements around the crack, including the quarter point element. This has been exploited recently in [13].

6 ACKNOWLEDGMENTS

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