On the energy release rate of finite cracked bodies

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Abstract

A new concept of the energy release rate of a finite cracked body is proposed. Considering the global view of the strain energy density field, the new fracture parameter presented here is different from the conventional definition that only depends on the stress field around the crack tip but neglects the influences induced by the boundary conditions on the far field. Based on the hypothesis of the energy density theory, the fracture initiation, trajectory, and destination can be predicted from the strain energy density field of the finite cracked body. The new energy release rate defined here is the integration of the strain energy density along the fracture trajectory that begins at the crack initiation and ends to the cracking destination point. Moreover, some interesting comparisons between the new and the conventional energy release rate are discussed.

Keywords Energy release rate; Crack; Energy density theory

1. Introduction

The energy density theory (Sih and Chen, 1973; Sih, 1974) was proposed as a fracture criterion that provides an alternative approach to failure prediction. This theory possesses the inherent advantage of being able to treat all mixed mode crack extension problems. Unlike the conventional energy release rate and stress intensity factor that measure only the amplitude of the local stress, the energy density factor, the fundamental parameter in this theory, defined as the coefficient of $1/r$ singular behavior of the volume strain energy density dw/dv is direction sensitive. The energy density theory has been widely used in the fracture communities and found in the literatures.

In this work, based on the energy density theory, a new concept of the energy release rate of a finite cracked body is discussed. The new parameter is defined as the integration of the strain energy density along the fracture trajectory that begins at the crack initiation and ends to the crack termination. In order to demonstrate the use of the present proposed parameter, numerical examples of zirconia and aluminum alloy are discussed in detail and shown in graphic form.

2. Energy release rate

Based on the strain energy density theory, we now propose to define a new energy release rate as

$$G_f = \int_{\lambda}^{g} \left(\frac{dw}{dv} \right)_{\min}^{\max} dL \tag{1}$$

where $(dw/dv)_{\min}^{\max}$ indicates the max. of $(dw/dv)_{\min}$, λ is the local location of max. of $(dw/dv)_{\min}$ on the outer core region, g is the global location of max. of $(dw/dv)_{\min}$, and L is the path of max. of $(dw/dv)_{\min}$, as shown in Fig. 2. The energy release rate G_f defined in (1) represents an explicit concept that integrates the strain energy density from the crack initiation, along the crack propagation, and to the crack termination. Once the crack begins to grow, the crack initiation occurs at the $(dw/dv)_{\min}$ on the border of the core region, i.e., at λ, so λ is

the location of the crack initiation. The crack will propagate along the path of $(dw/dv)_{\min}$, denotes as L, and then grows to the global $(dw/dv)_{\min}$ point, denotes as g. The location of g point can be regarded as the terminal point of the crack growth, at which the stress becomes to compression.

3. Numerical results and discussions

Referring to Fig. 1, a finite cracked plate subjected to a uniform stress σ_0 is considered. The elastic solution can be obtained by formulating the Hilbert problem in conjunction with the boundary collocation method as discussed in Section 2. In this work, two materials are discussed, which are zirconia and aluminum alloy, of which the fracture parameters are listed in Table 1. Figs. 3 and 4 show the difference between the conventional energy release rate G and the new defined energy release rate G_f with various b/w ratios. For zirconia, Fig. 3 illustrates that the difference between the two parameters G and G_f is getting larger for small b/w and getting closer as b/w increases. For aluminum alloy, Fig. 4 shows there is a certain gap between the two parameters and the difference between them is getting large as b/w decreases. From Figs. 3 and 4, they clearly show the obvious difference between G and G_f. It is seen that the difference between G and G_f of aluminum alloy is larger than that of zirconia, because the core region r_c of aluminum alloy is larger than that of zirconia, referring to Table 1. It means that aluminum alloy is more ductile than zirconia since its l point locates farther from the crack tip. Moreover, the g point locates at the outer boundary when b/w is large, which means that the rapid crack propagation prevails and the crack will grow immediately to the outer boundary. However, the difference between them becomes pronounced when b/w decreases, because the g point moves gradually into the inner domain. Therefore, the results show that the conventional energy release rate G is a limiting

case of the new defined energy release rate G_f where the l locates at the crack tip, i.e., $r_c = 0$, and g locates at the outer boundary. It also emphasizes that the new defined energy release rate G_f provides a comprehensive fracture parameter in consideration of both the local and global viewpoint of the cracked body and modifies the lack of the conventionally defined parameter G that is restricted to the brittle fracture and self-similar fracture behavior.

References

1. Sih, G.C., and Chen, E.P., 1973, Fracture analysis of unidirectional composites, Journal of Composite Materials 7, pp. 230-244.
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Table 1 Fracture parameters of zirconia and aluminum alloy

	$\sigma_u(\times 10^6 Nm^{-2})$	$K_{Ic}(\times 10^6 Nm^{-3/2})$	$G_c(\times 10^3 Nm^{-1})$	$r_c(\times 10^{-3} m)$	$S_c(Nm^{-1})$
zirconia	700	10	0.47	0.043	52.9
aluminum alloy	545	29.7	11.2	0.43	630

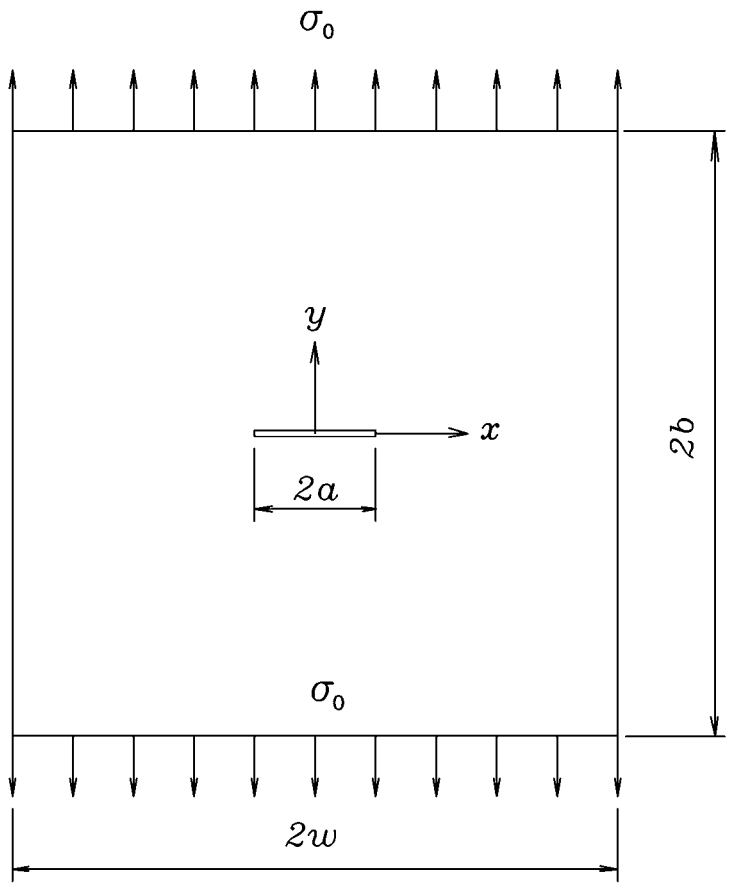

Fig. 1 The finite cracked body

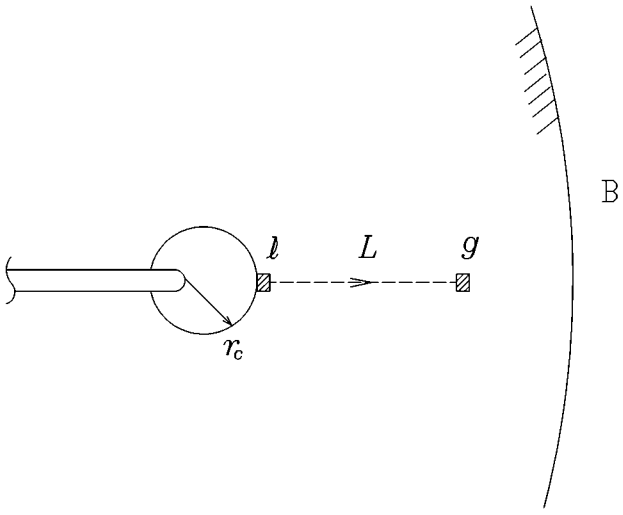

Fig. 2 Crack initiation, path, and termination

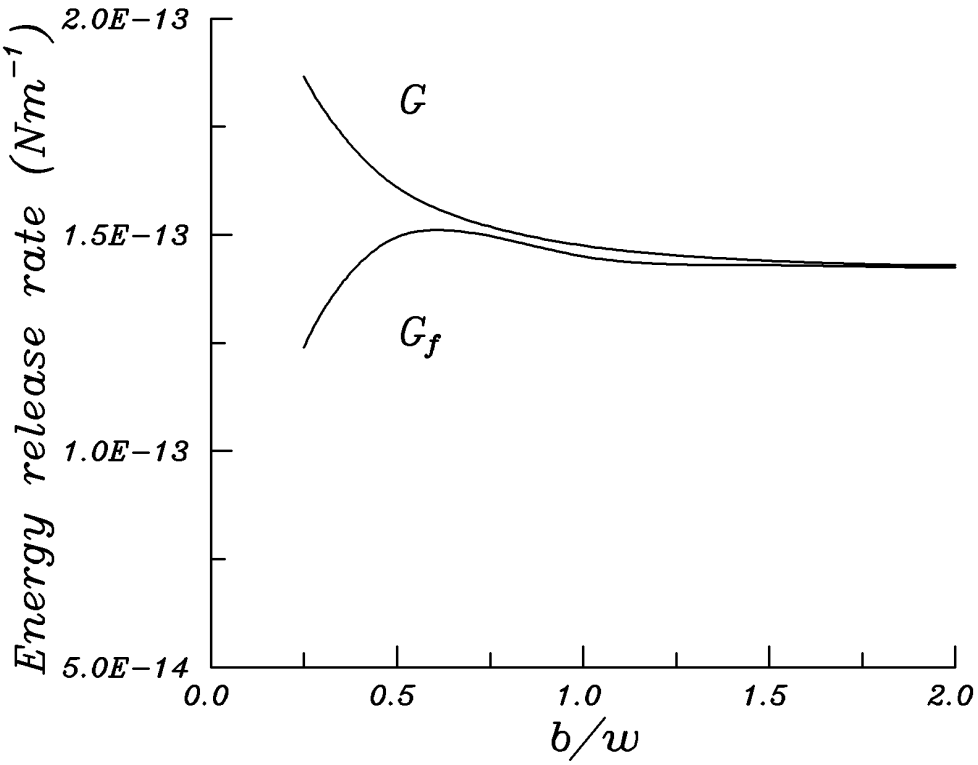

Fig. 3 Energy release rate of zirconia

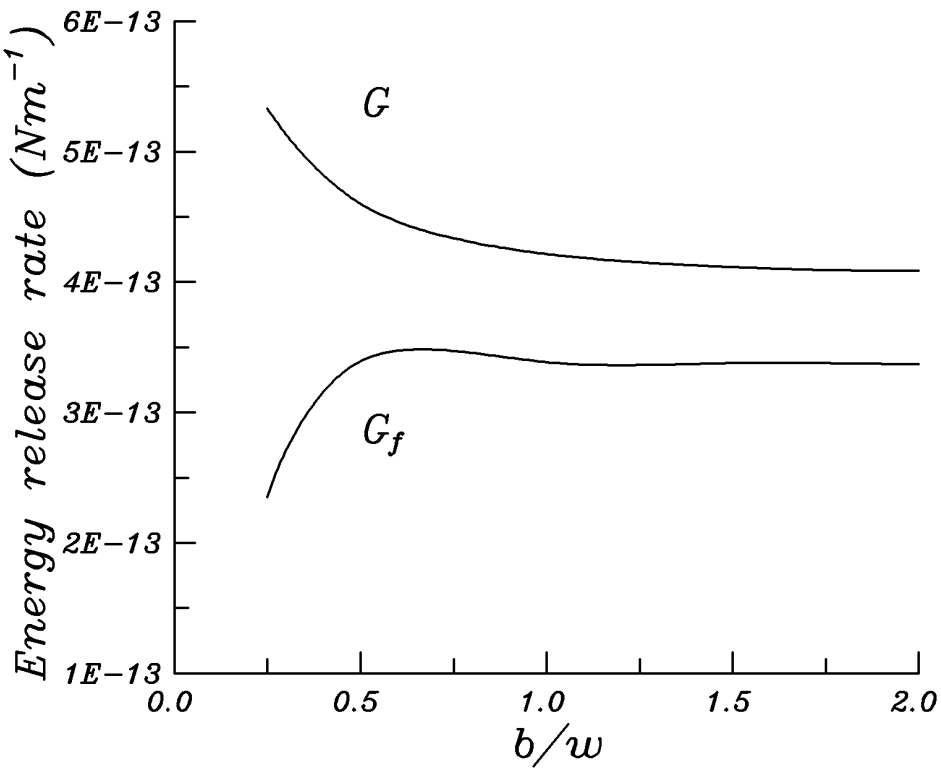

Fig. 4 Energy release rate of aluminum alloy