

# SIMPLE METHOD FOR ESTIMATION OF FATIGUE LOADING AND USEFUL LIFE FROM FRACTURE MORPHOLOGY

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## ABSTRACT

The paper presents a new method for a retro-estimation of the stress amplitude, the cyclic ratio and the number of cycles to failure from the fracture morphology of failed structural components. The method is based on a previously proposed procedure using a new relation enabling to describe the whole range of the long crack growth. The method is suitable for quantitative assessment of fatigue loading parameters and the number of cycles spent during the stable propagation. The following materials and fracture parameters are to be known: the v-K curve for the component material, mean projective striation spacing at least for one crack length and the location of the stable-unstable crack growth transition on the fracture surface. The shape functions of stress intensity factors related to decisive fatigue crack growth stages are to be anticipated as well. The application possibilities of the method are demonstrated in a case study concerning the fatigue failure of a compressor blade made from Ti-4Al-3Mo-1.5Zr high-strength alloy. Under the assumption of a nearly constant amplitude of applied loading, the analysis yielded the stress range  $\Delta\sigma \approx 550$  MPa, cyclic ratio  $R \approx 0.3$  and the number of cycles  $N_c \approx 10^6$  to propagate the initial 0.5 mm crack to its critical distance of 17 mm. These results appeared to be very useful for an explanation of the damage process occurred during the service life of the blade. The method can be also applied in case where a stationary random loading is to be expected. A special software package was also developed for university courses in applied fracture mechanics.

## 1 INTRODUCTION

The linear-elastic fracture mechanics can be successfully applied in numerical procedures predicting the residual fatigue life of structural components containing cracks longer than about 1mm. According to defect-tolerant design approaches to fatigue, the useful fatigue life is the number of cycles to propagate the largest undetected crack to an unstable fracture. In failure analyses, the re-estimation of the stress amplitude, the cyclic ratio and the number of cycles to failure is of a main practical interest. First attempts to assess those parameters from the fracture surface, by assuming constant applied stress amplitude, were published in the early 1960s [1,2]. Since that time some simple analytical methods appeared as suitable for practical purposes even for cases of stationary random loading [3-6]. Nowadays, these methods became even more effective when supported by advanced computers. Unfortunately, time consuming modern numerical methods combined with a detailed quantitative fractography usually do not yield much more accurate results. The reason lies in a lack of knowledge in appropriate fracture mechanical description with respect to the complexity of applied loading spectra, striation configurations and fatigue crack paths in structural components [7,8].

The main aim of this paper is to present a new method based on the procedure previously proposed by Pokluda and Staněk [6] but using a new relation enabling to describe the whole range of the long crack growth, including the influence of cyclic ratio  $R$ . The method is suitable for quantitative assessment of fatigue loading parameters and the number of cycles spent during the stable propagation. The application possibilities of the method will be demonstrated in a case study concerning the fatigue failure of a compressor blade.

## 2 THEORETICAL BACKGROUND

It is well known that the final stages of fatigue life, i.e. the subcritical crack propagation and the unstable fracture, are usually reflected on the fracture surface in a characteristic way [9]. From a great number of macro- and micromorphological features, the following ones are most important for our further considerations:

- [i] An approximate 1:1 relation between the mean projected striation spacing  $\bar{d}$  and the crack growth rate in the Paris-Erdogan region of the stable crack growth.
- [ii] The morphological boundary between stable/unstable crack growth is associated with the the crack length  $a_c$  related to the cyclic fracture toughness  $K_c$  in the Forman et al. relation.

The basic crack-growth rate equation used in this analysis is that of Forman et al. [10], modified for better accommodation to the near-threshold region:

$$\frac{da}{dN} = A \frac{\Delta K^n - \Delta K_{th}^n (1-R)^{nm}}{K_c (1-R) - \Delta K}, \quad (1)$$

where  $\Delta K$  is the range of applied stress intensity factor,  $\Delta K_{th}$  is the threshold factor for  $R=0$ ,  $R$  is the cyclic ratio and  $A, m, n$  are experimental constants. Eqn (1) reduces to the Klesnil-Lukas relation [11] for  $\Delta K_{th} \approx \Delta K \ll K_c$ . As a rule,  $n \in (3, 5)$  and  $m \in (0.3, 0.5)$  for metallic materials.

The left-hand side of eqn (1) may be substituted by  $\bar{d}$  measured within the range of  $a/a_c \in (0.3, 0.9)$  on the fracture surface, usually well corresponding to the Paris-Erdogan region.

The value  $\bar{d}$  is to be determined as

$$\bar{d} = \frac{1}{k} \sum_{i=1}^k d_i \cos \alpha_i,$$

where  $\alpha$  is the angle between local- and average (macroscopic) growth rate vectors measured in  $k$  points corresponding to the same crack length  $a$  on the fracture surface. The moment of unstable fracture corresponds to the critical crack length  $a = a_c$  so that one can assume

$$K_c = \frac{\Delta K(a_c)}{1-R}. \quad (2)$$

Since  $\Delta K = \Delta \sigma \sqrt{\pi a} f(a/W)$ ,  $W$  is the width of the component, a numerical solution of eqns (1) and (2) yields the estimation of loading parameters  $\Delta \sigma$  and  $R$  applied during the fatigue failure of the investigated structural component. It should be noted that extensive handbooks of stress intensity factors are easily available today (e.g. Murakami [12]).

Substituting obtained values  $\Delta \sigma$  and  $R$  into the relation

$$N_c = \int_0^{N_c} dN = \frac{1}{A} \int_{a_0}^{a_c} \frac{K_c (1-R) - \Delta K}{\Delta K^n - \Delta K_{th}^n (1-R)^m} da \quad (3)$$

one can calculate the number of cycles  $N_c$  associated with the stable propagation of fatigue crack in the length range of  $a \in \langle a_0, a_c \rangle$ , where  $a_0$  is the initial crack length of about 1 mm (or a minimum detectable length of inspection methods).



Figure 1: Bottom part of the fractured blade.

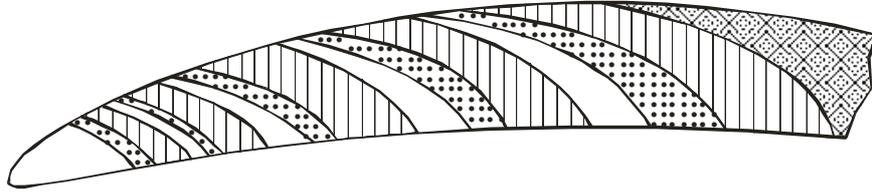


Figure 2: Scheme of macromorphological features on the fracture surface of the blade.

Thus, the following steps are to be performed to accomplish the failure analysis:

- [i] Experimental determination of eqn (1) by using samples made from the material of the fractured component.
- [ii] Application of appropriate shape function(s) of  $K$ -factor related to crack lengths  $a$  and  $a_c$ .
- [iii] Measurement of  $\bar{d}$  for the chosen length(s)  $a$  on the fracture surface.
- [iv] Measurement of the critical length  $a_c$  on the fracture surface.
- [v] Numerical solution of eqns (1), (2) to estimate  $\Delta\sigma$  and  $R$ .
- [vi] Anticipation of the shape function  $f(a/W)$  in the whole range of  $a \in \langle a_0, a_c \rangle$ .
- [vii] Numerical integration of eqn (3) to assess  $N_c$ .

When a fatigue loading of highly variable  $\Delta K$  but of a nearly constant  $R$  (e.g. the stationary random loading) is to be presumed, striation spacings  $d_1$  and  $d_2$  for two different crack lengths  $a_1$  and  $a_2$  are to be measured on the fracture surface [6]. The reason lies in an uncertain stress range causing the final fracture at the crack length  $a_c$ . In that case, the value of the stress range obtained by a simultaneous numerical solution of corresponding eqns (1) has a meaning of the root mean square value  $\Delta\sigma_{rms}$  [6,13].

### 3 APPLICATION EXAMPLE

The method was used to reconstruct conditions of fatigue failure of a compressor blade in an aircraft engine after a general repair (see Fig. 1). The engine was tested in a stand placed on the ground inside a special semi-natural cave. Sequences consisting of several blocks of constant loading amplitude were applied by changing engine frequencies to simulate the service regime. The blade was made from Ti-4Al-3Mo-1.5Zr high-strength alloy of the yield strength  $\sigma_y = 1060$  MPa. A careful investigation in the scanning electron microscope revealed that the fatigue crack was initiated at the entering edge of the blade as a consequence of impacting silicon microparticles. Most probably, these particles came from a dust whirled up from a poorly cleaned ground floor in the period of about a week.

The appropriate shape function for the cracked compressor blade

$$f(a) = 0.56419 \cdot (1 - 1.1 \cdot 10^{-4} a^3) \cdot (1.67687 - 0.43573a + 0.0819338a^2 - 0.0065158a^3 + 0.00018858a^4)$$

was obtained by compliance measurements performed on the blade containing crack growing from the entering edge [14]. The influence of a variable blade width  $W$  is already respected by coefficients in the shape function ( $a$  in [mm]). The macromorphology of fracture surfaces exhibited a sequence of characteristic regions and growth curves corresponding to different

Table 1: Measured and calculated values

$a_s$ [mm]	$d$ [nm]	$R$	$\Delta\sigma$ [MPa]	$N_c$
8	45	0.345	518	$2.01 \cdot 10^6$
10	120	0.265	582	$1.28 \cdot 10^6$

loading blocks (see Fig.2). Biode-carbon replicas of fracture surfaces had to be prepared for observations using the transmission electron microscope in order to find some facets covered by fatigue striations. An example of striation field, discovered in such way, is shown in Fig. 3. As expected, the striation spacing within one facet is nearly uniform. The critical crack length  $a_c = 17$  mm was found to correspond to the stable/unstable boundary (see also Fig.2). Three-point bending specimens made from the blade material were used for measuring parameters in eqn (1):  $\Delta K_{th} = 5$  MPa ,  $K_c = 80$  MPa ,  $A = 4.5819 \cdot 10^{-36}$  [Pa, m],  $n = 4.9$  and  $m = 0.3$ . Measured mean striation spacing  $\bar{d}$  related to two different  $a$  is shown in Table 1 together with calculated values of  $R$ ,  $\Delta\sigma$  and  $N_c$ .



Figure 3: Fatigue striations in the biode-carbon replica of the fracture surface ( $a = 10$  mm, 10000x)

Computed values of  $\Delta\sigma \approx 550$  MPa lie below the fatigue limit  $\sigma_c \approx 800$  MPa ( $R \approx 0.3$ ) of the blade material. Also the residual fatigue life  $N_c \approx 10^6$  cycles (from the initial crack length  $a_0 = 0.5$  mm) shows that the stable crack propagation run under the stress below the fatigue limit. Assuming the vibration frequency of several hundred Hz, the duration of the whole fatigue process could not overcome several days. Those results are consistent with the assumption of severe environmental conditions ruled in the semi-natural cave during the critical week.

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### References

- [1] Forsyth P. J. E. and Ryder D. A.: Aircraft Engineering 32 (1960), 96.
- [2] Jacoby G.: Experimental Mechanics 5 (1965), 65.
- [3] Peel C. J.: An Analysis of a Test Fatigue Failure by Fractography and Fracture Mechanics. Royal Aircraft Establishment TR 72034, 1972.
- [4] Ryder D. A. and Bunk W.: Zeitschrift für Flugwissenschaften 22 (1974), 223.
- [5] Uchimoto T., Sakamoto A. and Nagata S.: Transactions ISIJ 17 (1977), 1.
- [6] Pokluda J. and Staněk P.: Acta Technica ČSAV 4 (1981), 415.
- [7] Suresh S.: Fatigue of Materials. Cambridge University Press, 1998.
- [8] Pook L. P.: Crack Paths. Wit Press, 2002.
- [9] Mills K. et al.: ASM Handbook, Vol. 12, Fractography. ASM International 1992.
- [10] Forman R. G., Kearney V. E. and Engle R. M.: Transactions ASME 3 (1967), 459.
- [11] Klesnil M and Lukáš P.: Engineering Fracture Mechanics 4 (1972), 77.
- [12] Murakami Y.: Stress Intensity Factors Handbook. Pergamon Press 1992.
- [13] Barsom J. M.: ASTM STP 595 (1976), 236.
- [14] Prokopenko A. V.: Problemy prochnosti 4 (1981), 105 (in Russian).