TIME DISCONTINUOUS GALERKIN METHOD FOR DYNAMIC CRACK GROWTH USING X-FEM

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ABSTRACT

In the past few years, lots of techniques were developped for modeling dynamic crack growth. One of the main difficulties is that the discretization of the problem is time dependent. This can produce numerical instabilities, uncontrolled energy tranferts and high frequency oscillations in the solution due to time discontinuities in the numerical model. This paper proposes, in the framework of the eXtended Finite Element Method (X-FEM), a study of Time Discontinuous Galerkin Method (T-DGM). Combining efficient tools like X-FEM and T-DGM , the obtained results are well accurate and will allow to check efficiency for crack initiation, growth and arrest criteria.

1. INTRODUCTION

In the past few years, lots of techniques were developped for modeling dynamic crack growth : EFG (see Krysl [1], Organ [2]), GEM (see Duarte [3]) and various PUM (see Remmers [4], Belytschko [5]). One of the main difficulties is that the discretization of the problem is time dependent. This can produce numerical instabilities, uncontrolled energy tranferts and high frequency oscillations in the solution due to time discontinuities in the numerical model. Authors have proposed an enrichment strategy for X-FEM (Réthoré [6]) that allows stable numerical simulation and energy preservation. The results they obtained let think that the limits of classical Newmark time scheme are reached. It means that high frequency oscillations appears in the solution because such time integrator are not well appropriate for time discontinuities. This paper proposes, in the framework of the eXtended Finite Element Method (X-FEM), a study of Time Discontinuous Galerkin Method (T-DGM) (see for e.g Michler [7] or Li [8]). We first describe the space discretization of the model and then the time integration. The results are compared with those obtained using a Newmark type scheme and seem promising.

2. SPACE DISCRETIZATION

In the method presented, we use the eXtended Finite Element Method first introduced in Black [9]. In this method, an enrichment is added to the classical finite element approximation using the PUM developped in Babuska [10]. For static problem, the displacement field can be written with enriched basis of shape functions :

$$U = \sum_{i \in \mathcal{N}} N_i(x) U_i + \sum_{i \in \mathcal{N}_{cut}} N_i(x) H(x) a_i + \sum_{i \in \mathcal{N}_{branch}} \sum_{\alpha} N_i(x) B_\alpha(x) b_{i,\alpha} \tag{1}$$

where $\mathcal{N}$ is the set of all nodes in the mesh, $\mathcal{N}_{cut}$ the set of nodes which belong to elements completely cut by the crack and $\mathcal{N}_{branch}$ the set of nodes which belong to elements partially cut by the crack. N_i are the classical shape functions, H is a discontinuous function which value is

1 if x is above the crack surface and -1 if x is bellow. $[B_\alpha]$ are branch functions ((r,θ) are local cylindrical crack tip coordinates):

$$[B_\alpha] = \left[\sqrt{r} sin(\frac{\theta}{2}), \sqrt{r} cos(\frac{\theta}{2}), \sqrt{r} sin(\frac{\theta}{2}) sin(\theta), \sqrt{r} cos(\frac{\theta}{2}) sin(\theta) \right] \tag{2}$$

In our approach, all fields (displacement, velocity and acceleration) are discretized with Equation (1). Consequently, the discrete enriched problem can be written as a classical dynamic problem in the scope of the Finite Element Method : we define the mass matrix and the stiffness matrix in the usual manner. New space shape functions are added to simulate the crack growth. As shown by Figure 1, new singular enrichment is added on the new set $\mathcal{N}^{n+1}_{branch}$. For discontinous enrichment, new shape functions are only added on the set $\mathcal{N}^{new}_{cut} = \mathcal{N}^{n+1}_{cut} \setminus \mathcal{N}^{n}_{cut}$. This strategy is stable and energy preserving (see Réthoré [6]) refering to theoretical studies of Newmark type schemes for time dependent discretization (see Réthoré [11]).

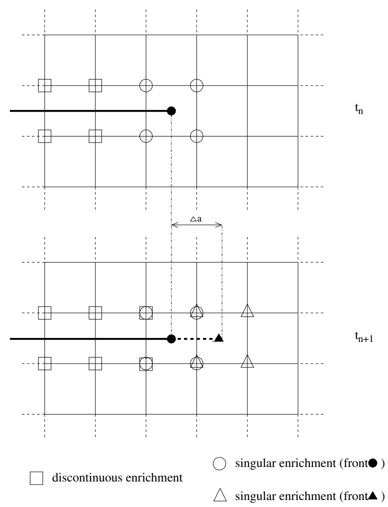

Figure 1. Strategy for enrichment when the crack propagates

3. TIME INTEGRATION

The formulation we choose is from Michler [7] and generalized for X-FEM space discretization. As soon as the formulation is velocity-based, the velocity is interpolated using a piecewise-linear approximation. Inside a time-slab $I_n = [t_n^+; t_{n+1}^-]$ we have :

$$V(t) = V_n \lambda_n(t) + V_{n+1} \lambda_{n+1}(t) \tag{3}$$

where λ_i are linear functions. The kinematic relation $\dot{U} = V$ and displacement continuity at t_n are strongly enforced. Velocity continuity at t_n is weakly enforced and the variational statement for I_n is written: find a linear function V such that for all linear test function δV we have:

$$\int_{I_n} \delta V(t) \left[M\dot{V}(t) + K \left(U(t_n^-) + \int_{t_n^+}^{t} V(\tau) d\tau \right) - F_{ext}(t) \right] dt + \delta V(t_n^+) \left(V(t_n^+) - V(t_n^-) \right) = 0 \tag{4}$$

This formulation is quasi-equivalent to the classical P3-P1 presented in Li[8] : computed values of velocity and displacement at the discrete instant are the same and the displacement continuity is ensured. The difference concerns the displacement interpolation and the kinematic relation : here the displacement approximation is P2 because the kinematic relation is strongly enforced, P3-P1 approximation allowed high order approximation but the kinematic relation is only weakly enforced.

4. EXAMPLE

The example we choose is the infinite plate with a semi-infinite crack because the theoretical solution is known (see Freund [12]). Several authors have already treated this example so we can also compare our results to their works. Under those assumptions (infinite plate with a semi-infinite crack), for the geometry described Fig. (2), the analytical solution is only valid for time $t \leq 3t_c = 3H/c_d$ when the reflected stress wave arrives on the crack tip. As the wave reaches the crack, the mode I stress intensity factor can be written for a moving crack :

$$K_1^{dyn}(\dot{a}, t) = \frac{2\sigma_0}{1-\mu}\sqrt{\frac{c_d t(1-2\mu)}{\pi}}\,\frac{1-\dfrac{\dot{a}}{c_r}}{1-\dfrac{\dot{a}}{2c_r}} \tag{5}$$

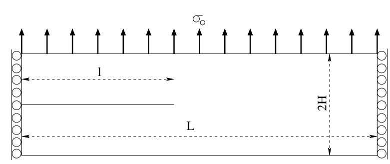

Figure 2. Geometry and loading for the example of the infinite plate

The numerical results are compared with this theoretical one. We will be interested in the case where the crack propagates at a prescribed constant speed v_0 after time $t = 1.5t_c$. The following numerical results are obtained with $H = 2m$, $L = 10m$ and $l = 5m$ for plate dimensions and $E = 210GPa$, $\nu = 0.3$ and $\rho = 8000kgm^{-3}$ for material properties. The tensile stress σ_0 is $500MPa$. v_0 is $1500m.s^{-1}$ and stress intensity factors are normalized by $\sigma_0\sqrt{H}$. Solutions are computed using a 40×80 quadrangle elements with linear approximation.

Results are presented on Figure 3. As shown in Michler [7], the convergence of the T-DGM formulation allowed us to use an approximately four time larger time step for the same accuracy. Using a Newmark type scheme (average acceleration method, $\gamma = \frac{1}{2}$, $\beta = \frac{1}{4}$), the slope is well captured but oscillations appear when the crack is growing. Such oscillations are not observed with the T-DGM and the solution remains satisfying.

5. CONCLUSION

This paper presents a combination of X-FEM and T-DGM for dynamic crack propagation problem. We propose an enrichment strategy for time dependent problems with X-FEM which is stable and energy preserving (see Réthoré [6]). The time integration is treated with T-DGM using a velocity-based formulation which ensures displacement continuity at the discrete instant. The results are compared with Newmark time integrator which seems to reach its limits for such time discontinuous problems. Combining efficient tools like X-FEM and T-DGM , the obtained results are accurate and will allow to check efficiency for crack initiation, growth and arrest criteria.

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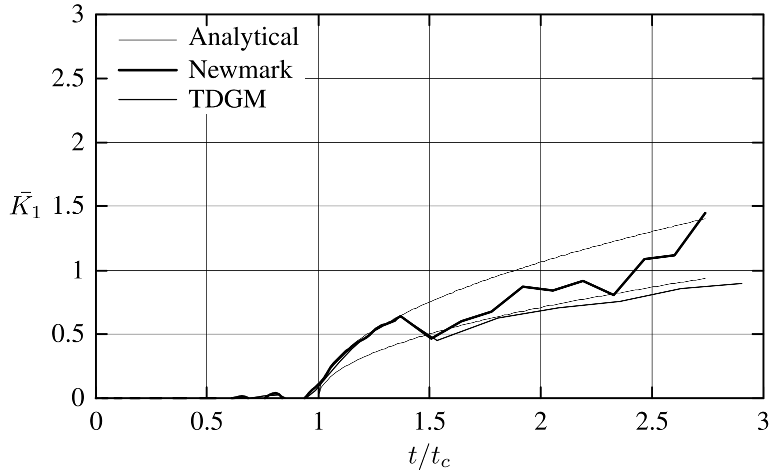

Figure 3. Numerical and analytical solutions $\bar{K}_1$ for a stationary then moving crack

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