DISCRETE MODELS OF INTERFACE DAMAGE

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ABSTRACT

An alternative to the continuous and deterministic modeling of continuum damage mechanics consists in a probabilistic approach: this is generally achieved for brittle materials by the weakest link model [1] and its generalizations, [2]. Discrete probabilistic models of interface damage are herewith elaborated, that extend the fibber bundle Daniel's model [3] to the consideration of different rupture probability laws and to a viscous behaviour of the interface.
We consider a fibre bundle, consisting of a finite number of parallel fibbers equally stretched between two rigid beams. The threshold traction force for the rupture of the fibbers is selected as a random variable. The bundle is connected with a testing machine, considered as an elastic solid having a known stiffness k. The displacement u of the interface varies between 0 – unloaded interface - to unity, which corresponds to the interface failure. For a distribution of the rupture threshold that obeys Weibull's law (with parameter α), it is shown that the bifurcation behaviour occurs if the stiffness k is less than a limit value $k(\alpha)$. A viscous behavior of the interface is further considered: the rheology of the interface is described by a bundle of viscoelastic fibers, selecting a Kelvin Voigt scheme. A damage variable is defined as the proportion of broken fibbers; the constitutive law of the bundle then relates the effective stress acting on the interface to the strain rate. Considering a creep problem, the strain response versus time obtained for a uniform damage law shows the three stages of creep, above a critical value of the applied stress (below this value, the deformation evolves monotonically towards an asymptotic value). A recursive model of the fibber bundle is next considered, and the strain response in a creep problem is calculated.

1. INTRODUCTION

We consider the following model system: a finite number N of parallel fibres are equally stretched between two rigid beams, representing a fibre bundle. The threshold traction force for the rupture of the fibbers is selected as a random variable. In order to emphasise the probabilistic aspects, the stiffness of all the fibres is taken equal to unity. The bundle is connected with a testing machine, considered as an elastic solid having a known stiffness k. The mean force, which is the applied force divided by the total number of fibbers N, is evaluated as $F(u) = (1 - P(u))u$, with

$P(u)$ the cumulative distribution function. The displacement u of the interface varies between 0 – unloaded interface - to unity, which corresponds to the interface failure.

2. WEIBULL'S STATISTICS OF THE RUPTURE FORCES

For a distribution of the rupture threshold that obeys Weibull's law (Weibull's law depends on two parameters (α, β): for convenience and without restricting the generality, we choose $\beta = 1$ in the sequel), the total displacement U and the force F applied to the bundle express successively as

$$U = u + \frac{u e^{-(u/\beta)^{\alpha}}}{k} \quad ; \qquad F = u e^{-(u/\beta)^{\alpha}}$$

The interface response is shown below (Fig. 1), in the case of a continuous behavior (equivalent to the asymptotic behavior of a 1000 fibbers bundle).

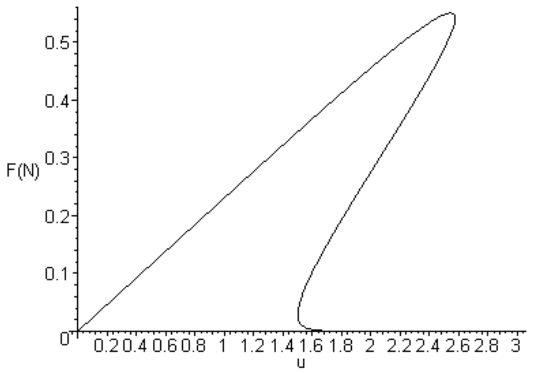

Fig. 1 : Weibull's law – continuous case - $\alpha = 4$

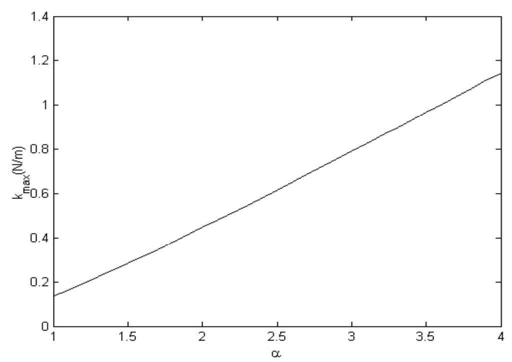

Fig. 2 : Maximum stiffness for bifurcation

A bifurcation is obtained when the maximum displacement is reached, $\frac{\partial U}{\partial u} = 0$. The condition for the existence of a bifurcation point thus resumes in finding the conditions to have real roots of the following equation, in which u is the unknown:

$$\frac{\partial U}{\partial u} = 1 + \frac{1}{k} e^{-u^{\alpha}} (1 - \alpha u^{\alpha}) = 0$$

The bifurcation occurs if the stiffness k is less than the obtained limit value $k(\alpha)$ (Fig. 2).

3. VISCOUS BEHAVIOUR OF THE INTERFACE

A viscous behaviour of the interface is further considered: the rheology of the interface is described by a bundle made of viscoelastic fibbers; selecting a Kelvin Voigt scheme, the relationship between stress and strain is accordingly given by

$$\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt}$$

where E is the Young modulus and η the viscosity of the interface material. The damage variable D is defined as the proportion of broken fibbers, thus the effective stress acting on the interface

$$\sigma := \frac{\Sigma}{1-D}$$

with Σ the nominal stress. Considering a creep problem, - Σ is constant - , the strain function $\varepsilon(t)$ satisfies the differential equation:

$$\frac{dt}{d\varepsilon} = \eta/E \Big/ \left[\frac{\Sigma}{E(1-D)} - \varepsilon\right]$$

that can either be solved analytically (in some cases, e.g. continuous damage) or numerically, in the case of discrete damage or for more sophisticated probability laws. Let investigate the case of continuous damage with a damage law $D(\varepsilon) = \varepsilon$, corresponding to the uniform law. For $\frac{\Sigma}{E} > \frac{1}{4}$, we obtain curves (time evolution of the total strain) presenting an inflexion point (solid line on Fig. 3 a) ; after that stage, damage accelerates, the system is no longer stable, and the interface breaks.

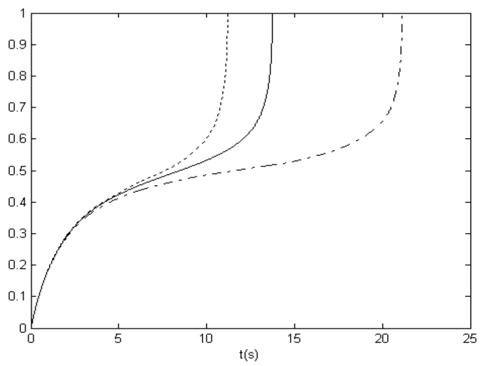

Fig. 3a: Evolution of strain vs. time – Continuous damage (solid line) vs. discrete damage realizations (dotted lines)

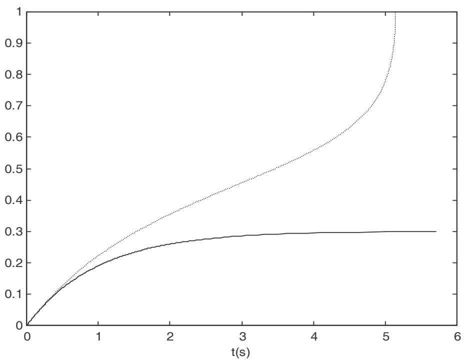

Fig 3b :Evolution of strain vs. time damaged interface (dotted line) virgin interface (solid line)

If $\frac{\Sigma}{E} < \frac{1}{4}$, ε stays below 1 while reaching an asymptotic value. This modeling allows taking into account the different stages of creep: primary creep (no damage), secondary creep, ternary creep and finally break. Secondary and ternary creeps are due to the apparition of micro cracks, which is well described by damage. The comparison between continuous and discrete damage shows a real dispersion (dotted lines on Fig. 3a) among the times before break, due to the probabilistic aspect of the modeling. Contrary to this, the case of a ramp of applied stress minimizes the differences between different realizations of the probability laws.

4. RECURSIVE MODEL OF THE VISCOELASTIC BUNDLE

Due to the presence and propagation of damage, a progressive degradation of the mechanical properties of the physical links between both surfaces will appear. One way of taking this degradation into account is to consider that the equivalent stiffness of the links decreases with damage [4]; this is being modeled giving recursive values to the Young modulus and the viscosity (coefficients α and β) (Fig.4).

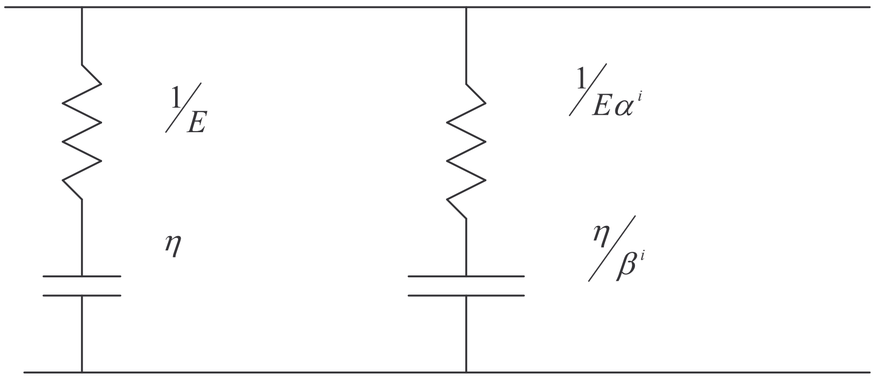

Fig. 4 : Discrete model of an interface endowed with recursive evolution of its properties

The constitutive law of the recursive block is evaluated by an electrical analogy, considering an equivalence between the inverse of the Young modulus with a resistor and the viscosity with a capacitor and fitting a Bode diagram

$$\sigma(t) = \frac{1}{\omega_0^n}\left(\frac{d}{dt}\right)^n \varepsilon(t)$$

with $\omega_0 = E/\eta$ and $n = \dfrac{1}{1+\dfrac{\log\beta}{\log\alpha}}$, which is a Scott Blair type behavior law [5]. Thus, the behavior law of the material has the following expression:

$$\sigma(t) = E\varepsilon(t) + \frac{1}{\omega_0^n}\left(\frac{d}{dt}\right)^n \varepsilon(t)$$

In a creep problem (the stress $\sigma(t)$ being assigned a fixed value Σ), an for an undamaged material, we have to solve the following equation:

$$\Sigma = E\varepsilon(t) + \frac{1}{\omega_0^n}\left(\frac{d}{dt}\right)^n \varepsilon(t)$$

The solution is given by the Laplace transform [6]. For example, if $n = 1/2$, we have

$$\frac{\Sigma}{s} = EE(s) + \frac{1}{\sqrt{\omega_0}}\sqrt{s}E(s) + \gamma$$

where $E(s)$ is the Laplace transform of $\varepsilon(t)$ and $\gamma = -\dfrac{1}{\sqrt{\omega_0}}\dfrac{d^{-1/2}\varepsilon}{dt^{-1/2}}\Big|t=0$. By an inversion of the Laplace transform, we get:

$$\varepsilon(t) = -\gamma\sqrt{\omega_0}\,t^{-1/2}E_{1/2,1/2}(-E\sqrt{\omega_0}\sqrt{t}) + \Sigma\sqrt{\omega_0}\int_0^t\left[u^{-1/2}E_{1/2,1/2}(-E\sqrt{\omega_0}\sqrt{u})\right]du$$

in which $E_{1/2,1/2}$ stands for a Mittag-Leffler function defined by $E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}$ for α and β positive. The shape of this solution (Fig. 5) resembles the curve given on Fig.3b for a virgin interface.

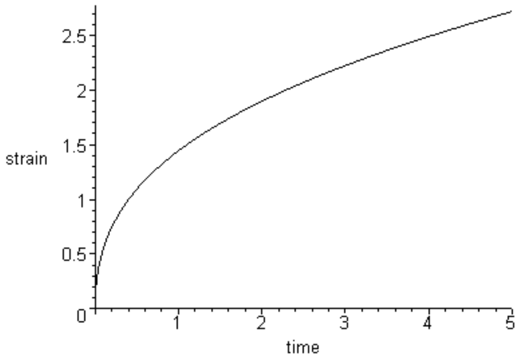

Figure 5: response of the interface obeying a Scott-Blair type behavior

In a traction test, with a ramp of applied stress $\Sigma=\sigma_0\frac{t}{\tau_f}$, the solution is given by

$$\varepsilon(t)=-\gamma\sqrt{\omega_0}\,t^{-1/2}E_{1/2,1/2}(-E\sqrt{\omega_0}\sqrt{t})+\frac{\sigma_0}{\tau_f}\sqrt{\omega_0}\int_0^t\left[(t-u)u^{-1/2}E_{1/2,1/2}(-E\sqrt{\omega_0}\sqrt{u})\right]du$$

The response of the interface is represented for $\gamma=0$ (fig. 6), in terms of the time-strain relationship. Note that for non nil values of γ, a singularity occurs at the initial time, due to the divergence of the Mittag Leffler function at that point.

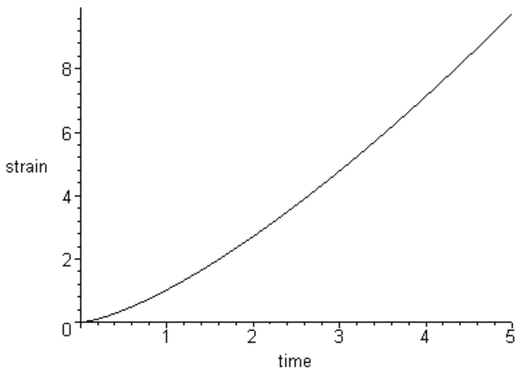

Figure 6: traction test: evolution of strain vs. time - Scott-Blair type behavior

This way of approaching the behavior of interfaces has further been extended to configurations representing adhesive bonding, and to the modeling of adhesion phenomena.

5. PERSPECTIVES

Amongst the perspectives, the importance of the time variable is one of the key aspects that shall further be investigated: in the probabilistic description of the behavior of the interface, the role of time is equivalent to the role of strength in a traction test for example. Processes certainly have a non-markovian character: the whole history of the damage has then to be taken into account in order to estimate the evolution of the system. A further step in the modeling of the topology of the discrete links is the consideration of the transverse coupling between the fibbers; as a consequence, it will certainly lead to a local redistribution of strengths in the bundle, with application to solids presenting internal defects like holes and cracks.

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