A MECHANISM OF FAILURE OF CONCRETE AND ROCK IN COMPRESSION

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ABSTRACT
We consider a mechanism of macroscopic crack growth and failure in cement and rock in compression based on three dimensional patterns of stress non-uniformity associated with generation of multitudes of wing cracks. In 3D each wing crack has a limiting ability to grow and hence cannot produce sample failure on its own. Neither the crack coalescence can form 3D patterns that can evolve into a macroscopic crack. Instead opening and shearing of the wing cracks produce additional stress disturbance. The combined effect of the stress disturbances from all wing cracks results in a non-uniform stress field spatially varying in a random fashion. The main feature of such a field is that any plane running through the sample can potentially have parts subjected to tensile stress alongside with the parts under compression (the average stress equal to the applied external load acting on this plane). As the load increases, these stress variations become stronger and, eventually, produce a macroscopic tensile crack at the place the tension was maximal. Further growth of the macrocrack proceeds by initiating new segments, offset from the main crack plane in order to avoid the places under compression. This en-echelon type fracture is formed through a specific mechanism of tensile crack growth rather than coalescence. The macroscopic crack is inclined to the direction of axial compression at the angle maximising the average magnitude of the tensile parts of the stress field. This angle depend upon the ratio between total normal opening (dilatancy) and shear of the wing cracks, which in its own term depends upon the material microstructure and the confining pressure. When this ratio is above a certain threshold, the macrocrack will be parallel to the direction of axial compression producing splitting. When the ratio is below the threshold, the macrocrack will be inclined and look like shear fracture.

1. INTRODUCTION

Failure of heterogeneous materials such as cementicious materials, rocks and ice in compression is characterised by two major modes (see Germanovich [1] and the literature cited there): (1) splitting or columnar failure, predominantly observed in uniaxial compression; (2) shear or oblique failure observed in triaxial compression and, often in uniaxial compression. In the latter case the sample is broken by what appears as shear cracks.

The most popular approach to describe shear failure is to use the Mohr-Coulomb theory or its various modifications, which adequately represent experimental data related to the oblique failure. In this theory, as well known, the direction of the future fracture is determined as the one at which the shear stress reaches the friction stress at the least load magnitude the latter being referred to as the compressive strength. The drawbacks of this theory are also well known. Firstly, it has a contradiction in itself since it is based on friction properties of a not yet existing interface. This immediately turns the Mohr-Coulomb criterion into an empirical one in which the friction parameters are treated as internal material parameters to be back calculated from the results of compressive tests. Subsequently, the application of the criterion becomes limited to the cases allowing direct testing, which often excludes in-situ characterisation since direct transfer of laboratory data to large-scale situations is precluded by the scale effect. The second drawback is the inability of the Mohr-Coulomb theory to explain the splitting. In view of these drawbacks a considerable effort was devoted to developing micromechanical models of failure.

The majority of models developed to explain splitting are based on the concept of wing crack – the crack generated by a local stress concentrator (a pre-existing share crack or pore or a certain type of grain contact) assuming that the wing crack can grow extensively at least under uniaxial compression as observed in 2D experiments (e.g., Brace [2], Horii [3]). The failure is attributed either to the growth of one of the wing cracks throughout the whole sample or to unstable crack growth caused by interaction (e.g., Ashby [4], Germanovich [5], Kemeny [6]). These 2D models fail however to recognise that the real three dimensional wing cracks have an intrinsic limitation to the growth preventing the wing elongation beyond the size of the initial sear crack even in the most favourable case of uniaxial compression.

Modelling of shear failure in compression, taking into account that the shear cracks do not propagate in their own plane, but rather kink, is based on considering various mechanisms of crack coalescence (e.g., Wittmann [7], Stavrogin [8]) or en-echelon formation (e.g., Horii [3], Schulson [9], Reches [10]). The main issue the en-echelon models face is the identification of the mechanism which would force all the cracks forming en-echelon to be oriented in the same direction. If this happened by chance, then even for a 2D model when each crack has a choice of only two orientations with probability of ½, the probability to find n cracks equally oriented in desired locations is 2^{1-n}. Suppose the cracks are 1 mm in length. Then, in order to produce a section of a sample of 5 cm long one needs 50 cracks in en-echelon. The probability to find such an arrangement, $2^{-49}\approx 10^{-15}$, is negligible. In 3D the probability reduces further, since the cracks forming en-echelon must be more or less coplanar and also more cracks is needed (in the above example 2500 cracks will be needed to form a macrocrack of 5x5 cm). Hence, the cracks forming en-echelon were not there initially, but rather were formed in the process of macrocrack propagation.

Direct finite element simulations of failure in heterogeneous materials are based on specifying failure criteria for each element (e.g., Zou [11]) which are essentially the same as observed in macroscopic samples. Therefore, the question of the failure criterion is simply shifted from macroscopic to microscopic scale without actually producing the relevant failure mechanism. Models treating the shear cracks as planes of strain localisation (e.g., Rudnicki [12]) suffer from the same problem: the material behaviour at the micro level should resemble the macroscopic behaviour the model is set to explain.

Dyskin [13] noticed that the wing cracks, create considerable stress non-uniformity (spatial stress fluctuations) with some places of the materials subjected to tensile stresses and therefore capable of generating tensile cracks. Based on this idea a 3D model of splitting crack formation and propagation was proposed. In this paper we extent this idea to model the formation and propagation of inclined tensile cracks which produce oblique (shear-like) failure.

2. MECHANISM OF CRACK PROPAGATION IN NON-UNIFORM STRESS FIELDS

The stress field generated by wing cracks and other heterogeneities is non-uniform and random owing to their random locations, orientations, shapes and dimensions. In the parts of the sample where the stress variations become tensile new cracks can be generated and grown to macrocracks [13]. Figure 1a, b explains a possible mechanism of tensile macrocrack formation and propagation. Figure 1a shows a possible realisation of random field of a normal stress component σ_{33}; the stress increasing from dark to white, such that the dark areas correspond to compression, while the white areas correspond to tension. For the illustration purpose, only a section parallel to the (x_2, x_3) plane is shown. Obviously, the first crack (crack 1) is generated at the area with the maximum tensile stress. This crack will propagate until it is arrested in the areas subjected to compression. As the applied load increases, so does the amplitude of the stress variations. Further propagation of crack 1 will however be prevented by similarly increased compression; instead it will generate a new crack (crack 2) at the place where the original stress distribution showed no compression. This will result is a discontinuous offset-type trajectory of crack growth, which in the real 3D case will look like the one shown in Figure 1b. Essentially, the crack segments will be situated at places where no compressive stresses acted. On average, the magnitude of these stresses is equal to the mathematical expectation of positive (tensile) values

$$\sigma_+ = \int \max(\sigma, 0) f(\sigma) d\sigma \tag{1}$$

where σ denotes the relevant stress component, $f(\sigma)$ is the probability density function.

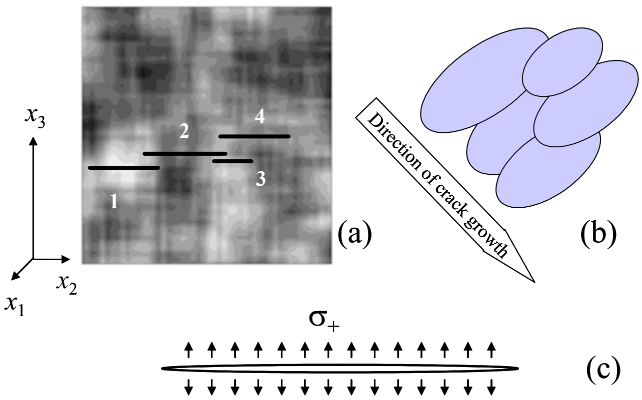

Figure 1: Macroscopic crack formation and growth under spatially random stress field: (a) a realisation of the random stress field σ_{33}; the maximum compression is shown in black, the tension - in white. The first crack segment (crack 1) appears at the place of maximum tension. Crack 2 is then generated at the place where the compressive stress is minimal, then other segments (cracks 3 and 4) are generated; (b) a 3D structure of the macrocrack; (c) a model of the macrocrack.

We will model such a complex macroscopic crack, in a very approximate manner, as a planar crack subjected to load σ_+, Figure 1c. Assuming further that the crack is disk-like of a radius R and using conventional criterion of crack propagation $K_I = K_{Ic}$, where K_{Ic} is the fracture toughness of the material, $K_I = 2\sigma_+(R/\pi)$, one notices unstable macrocrack propagation.

Suppose the random stress is Gaussian with the uniform mathematical expectation, σ_{av} and standard deviation, Σ. Then

$$\sigma_+ = \Sigma\left\{\xi\left[\frac{1}{2} - \Phi(-\xi)\right] + \frac{1}{\sqrt{2\pi}} e^{-\frac{\xi^2}{2}}\right\}, \quad \xi = \frac{\sigma_{av}}{\Sigma}, \quad \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{x^2}{2}} dx \tag{2}$$

3. STATISTICAL PROPRTIES OF NON-UNIFORM STRESS FIELD CREATED BY A MULTITUDE OF WING CRACKS

In order to quantify the mechanism by which this field produces and drives macrocracks we need to determine the mathematical expectation and variance of this field.

Suppose that the sample is such loaded that in a similar homogeneous sample a uniform stress field σ_{ik}^0 would be produced. In the case of compression of magnitude p in x_3 direction (see the co-ordinate frame on Figure 2) and confining pressure of magnitude q in the normal directions

$$\sigma_{11}{}^0 = \sigma_{22}{}^0 = -q, \quad \sigma_{33}{}^0 = -p, \quad \sigma_{12}{}^0 = \sigma_{13}{}^0 = \sigma_{21}{}^0 = 0 \tag{3}$$

The actual stress field σ_{ik} is different predominantly owing to the effect of wing cracks with some contribution from other heterogeneities. Nevertheless its volumetric average over the whole sample, $<\sigma_{ik}>$, can be shown to be equal to $\sigma_{ik}{}^0$. Assume ergodicity the mathematical expectation of this stress field can be found

$$\overline{\sigma_{ik}} = \langle \sigma_{ik} \rangle = \sigma_{ik}{}^0 \tag{4}$$

In order to estimate the variance of the stress field generated by the wing cracks each crack is modelled by a dislocation loop, Figure 2, with the shear component of the Burgers vector, b_t, directed parallel to the axial load and the normal component, b_n, directed perpendicular to the axial load. The shear component reflects the contribution of the wing crack to the non-linear part of axial strain, while the normal component reflects the wing crack contribution to dilatancy. As further simplification, we replace the dislocation loops with point defects, by limiting transition of the loop area, A, to zero keeping the corresponding volumes, $U_t=b_tA$, $U_n=b_nA$, constant.

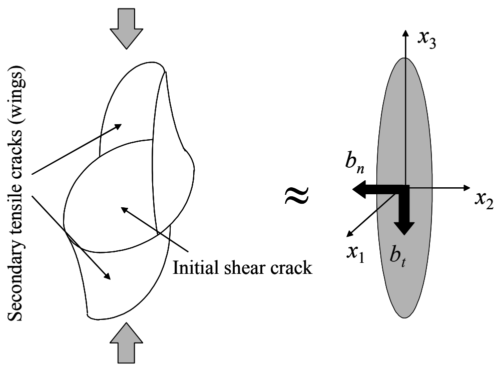

Figure 2: Wing crack evolved form an initial inclined shear crack (left) and its model as a dislocation loop (right) with Burgers vector (b_t,b_n).

We represent, following Landau [14], the dislocation through body forces $f_i=1/2\lambda_{iklm}((n_lb_m+n_mb_l)\delta(\zeta))_{,k}$, where n_m and b_m are the components of unit normal vector to the loop and the Burgers vector respectively, $\delta(\zeta)$ is the delta-function of coordinate ζ along the normal vector, $(_{,k})$ denotes differentiation with respect to x_k and summation is presumed over repeated indexes. Then using the divergence theorem the volumetric average of the stress field generated by these point defects can be expressed as follows

$$\left\langle \sigma_{ik}{}^{disl} \right\rangle = \sigma_{ik}{}^0 + \frac{E}{1+\nu}\frac{1}{V}\sum_{\mu=1}^{M}\left[U_n{}^{\mu}\left(n_i{}^{\mu} n_k{}^{\mu} + \frac{\nu}{1-2\nu}\delta_{ik} \right) + \frac{1}{2}U_t{}^{\mu}\left(n_i{}^{\mu}\delta_{k3} + n_k{}^{\mu}\delta_{i3} \right) \right] \tag{5}$$

where $\sigma_{ik}{}^0$ is the applied load, E, ν are the Young's modulus and Poisson's ratio of the material (the material is assumed isotropic), V is the sample volume, M is the number of wing cracks in the sample (this number grows with load), the superscript μ refers to a particular wing crack. It is important to distinguish between this stress field which essentially represents the stresses generated at a distance from the wing cracks (since this approximation relates to the scale from which the wing cracks are seen as point defects) with the full stress field (that includes stresses in immediate neighbourhoods of the wing cracks) which volumetric average is given by equation (4).

Direct computations of the correlation function for the stress fields in the point defect approximation [13] suggested that the correlation length is of the order of the wing crack size. Based on this observation, we break the sample volume V into M parts V_λ, $\lambda=1,..,M$ such that the averages over V_λ, $<\sigma_{ik}>_\lambda$ are approximately independent from each other. Then from the ergodicity, the variance $\mathrm{Var}(<\sigma_{ik}>_\lambda)$ can be expressed through the variance of the full volumetric average, $\mathrm{Var}(<\sigma_{ik}>)$. We assume that the latter is adequately represented by the variance of (5). For wing cracks uniformly oriented in the (x_1, x_2) plane, assuming that the average values of shear 'volume' U_t and volume of opening of wing cracks U_n are independent, one has

$$Var\langle\sigma_{ik}\rangle_\lambda = \frac{t^2}{2}\left[\kappa^2(1-\delta_{i3}\delta_{k3}) + (\delta_{i3}+\delta_{k3}-2\delta_{i3}\delta_{k3})\right], \quad t = \frac{NEU_t}{2(1+\nu)}, \quad \kappa t = \frac{NEU_n}{2(1+\nu)} \qquad (6)$$

where N is the number of wing cracks per unit volume, κ has the meaning of the ratio between dilatancy and inelastic part of the axial strain.

4. A MECHANISM OF SPLITTING AND OBLIQUE FAILURE IN COMPRESSION

Consider a plane inclined at an angle ψ to the x_3 axis and determine the average tensile stress σ_+ acting on that plane. Substituting (3), (6) into (2) one has

$$\sigma_+ = \frac{t}{2}\frac{1}{\sqrt{2\pi}}e^{-\frac{\xi^2}{2}}\cos^2\psi\sqrt{\kappa^2+2\tan^2\psi} - (p\sin^2\psi + q\cos^2\psi)\left[\frac{1}{2}-\Phi(-\xi)\right], \quad \xi = -\frac{2(q+p\tan^2\psi)}{t\sqrt{\kappa^2+2\tan^2\psi}} \qquad (7)$$

Figure 3 shows stress (7) for $t=p$ and for different q and κ. It is seen that for $\kappa=1$ stress σ_+ reaches maximum at $\psi=0$, which corresponds to splitting. Small values of κ lead to oblique failure. Since for $q>0$ mainly oblique failure is observed, κ should be small as compared to $\tan(\psi)$.Then taking into account that $t\sim p-q$ an expression similar to Coulomb-Mohr criterion can be obtained. If, in addition, $t>>p$ its parameters will become independent of the loads p and q.

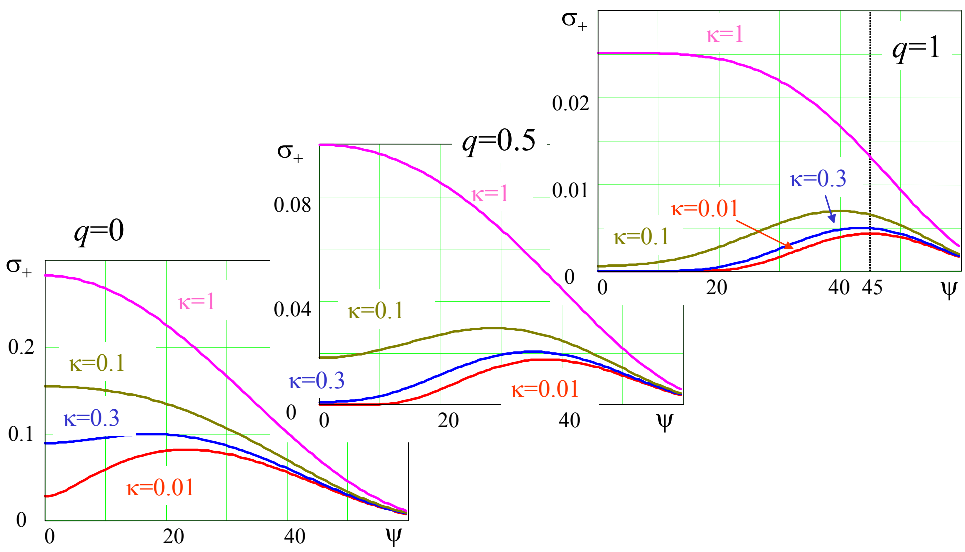

Figure 3: Dependence of average tensile stress acting on a plane vs. the angle of its inclination.

For uniaxial compression the critical value of κ has the form

$$\kappa_{cr} = -\frac{p\sqrt{2\pi}}{t} + \sqrt{\frac{2\pi p^2}{t^2} + 1} \qquad (8)$$

5. CONCLUSIONS

It is demonstrated that the random stress non-uniformity created by the multitude of wing cracks is sufficient to induce tensile cracks and then make them grow unstably as a macroscopic en-echelon tensile fracture. This macroscopic fracture is inclined to the direction of axial compression at the angle maximising the average magnitude of the tensile parts of the stress field. This angle depends upon the ratio between total normal opening (dilatancy) and shear of the wing cracks. When this ratio is above a certain threshold, the macrocrack will be parallel to the direction of axial compression producing splitting. When the ratio is below the threshold, the macrocrack will be inclined and look like shear fracture.

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