BRANCHING INSTABILITY OF BRITTLE FRACTURE

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ABSTRACT

A new method for determining the elastodynamic stress fields associated with the propagation of anti-plane branched cracks is presented. The exact dependence of the stress intensity factor just after branching is given as a function of the stress intensity factor just before branching, the branching angle and the instantaneous velocity of the crack tip. The jump in the dynamic energy release rate due to the branching process is also computed. Applying a growth criterion for a branched crack, it is shown that the minimum speed of the initial single crack that allows branching is equal to $0.39c$, where c is the shear wave speed. At the branching threshold, the corresponding bifurcated cracks start their propagation at a vanishing speed with a branching angle of approximately 40 degrees. Using these exact results, the branching of a single propagating crack under mode I loading is also considered and the critical velocity for branching is computed. Moreover, using the asymptotic expansion of the stress field ahead a curved extension of a crack, the experimentally observed shape of the branches is recovered without introducing any additional parameter. It is shown that the length scale introduced by the curved extension of the branches is the geometrical length scale of the experiment. The quantitative comparison with the experimental results agrees with the theoretical findings. The present study confirms that the continuum theory of brittle fracture mechanics provides the minimal ingredients for predicting the branching instability threshold, the branching angle and the subsequent paths of the branches.

1 INTRODUCTION

The field of fracture mechanics is concerned with the quantitative description of the mechanical state of a deformable body containing a crack or cracks. The continuum theory of fracture mechanics studies the nucleation of cracks, the conditions for which they propagate and their dynamics [1,2] In the framework of continuum theory of brittle fracture, the relationship between internal stress and deformation and the pertinent balance laws of physics dealing with mechanical quantities do not include the possibility of material separation. Indeed, the ``equation of motion'' of the crack tip is based on additional mathematical statements for crack growth. The most popular criterion for crack propagation consists of two parts; the Griffith postulate and the principal of local symmetry.

The Griffith energy criterion [3] states the intensity of the loading necessary to effectively promote propagation through $\boldsymbol{G}=\boldsymbol{\Gamma}$, where $\boldsymbol{G}$ is the energy release rate and $\boldsymbol{\Gamma}$ is the fracture energy. The principal of local symmetry states that the crack advances such that the shear stress in the vicinity of the crack tip vanishes; the loading there is purely tensile and pulls the crack faces apart. This rule was first proposed for slowly moving cracks [4], and generalized to rapidly moving cracks [5]. Moreover, it has been shown in [5] that the two criteria rise from the same physical origin. The energy release rate can be seen as the component, $\boldsymbol{F_1}$, of the driving force along the direction of crack motion. The Griffith energy criterion can thus be reinterpreted as a material force balance between $\boldsymbol{F_1}$ and a resistance force to crack advance per unit length of the crack front; $\boldsymbol{F_1}=\boldsymbol{\Gamma}$. However, this equation of motion is not sufficient to determine the trajectory of a crack if it is allowed to deviate from straight propagation. If one assumes that material force balance holds at the crack tip, one should impose that the component of the material force perpendicular to the direction of crack propagation also vanishes. This condition is identically satisfied if the loading in the vicinity of the crack tip is purely tensile.

The Griffith criterion combined with the principle of local symmetry predict adequately the path and the stability of slowly propagating cracks [6,7]. Controlled experiments on quasi-static cracks are not numerous, but they confirm the theoretical results [8,9]. In the case of fast crack propagation, an issue of great interest in crack dynamics is the question of understanding the instabilities associated with the appearance and evolution of kinked or bifurcated cracks. We are particularly interested in this phenomenon in brittle materials, in which several experiments [10-14] have shown evidence of these instabilities. The experiments on PMMA and glass samples [13,14] have identified a dynamic instability of a propagating crack, which is related to a transition from a single crack to a branched crack configuration. The instability occurs when the crack speed exceeds a critical velocity $\boldsymbol{v}_c$, which depends neither on the applied stress nor on the geometry of the plate. Above $\boldsymbol{v}_c$, a single crack is no longer stable. Instead, a repetitive process of micro-branching occurs, which changes the crack dynamics.

Given this experimental phenomenology, a theoretical approach that has been suggested is to describe kinked or bifurcated cracks in the framework of Linear Elastic Fracture Mechanics [15], which has proven its validity for single straight crack dynamics in brittle materials [1,2]. The main goal of this approach is to obtain expressions for the stress intensity factors at the crack tips of the emergent bifurcated cracks. The idea is to compare their associated energy release rates with the unperturbed configuration, i.e. a crack from which no branches emanated. The dependence of these energy release rates in the crack kink angles, velocities, etc.. may elucidate the origin of the instability. This line of work involves solving a dynamic linear elasticity problem where the crack boundaries develop a kinked shape that makes the theoretical analysis quite involved. Our final goal is to solve the problem of determining the stress intensity factors of a kinked crack that starts at a given angle and velocity from a main crack whose velocity is also given.

The aim of the present work is to present a new method to calculate the dynamic stress intensity factors associated with the propagation of anti-plane kinked or branched cracks [16,17]. This approach allows the exact calculation of the corresponding dynamic stress intensity factors. The latter are very important quantities in dynamic brittle fracture mechanics, since they determine the crack path and eventual branching instabilities. As an illustration, we consider the configuration of an initially propagating anti-plane crack that branches into two cracks that merge symmetrically. The explicit dependence of the stress intensity factor just after branching is given as a function of the stress intensity factor just before branching, the branching angle and the instantaneous velocity of the crack tip. The jump in the dynamic energy release rate due to the branching process is also computed. Similarly to the single crack case, a growth criterion for a branched crack is applied. It is based on the equality between the energy flux into each propagating tip and the surface energy that is added as a result of this propagation. It is shown that the minimum speed of the initial single crack that allows branching is equal to 0.39c, where c is the shear wave speed. At the branching threshold, the corresponding bifurcated cracks start their propagation at a vanishing speed with a branching angle of approximately 40 degrees.

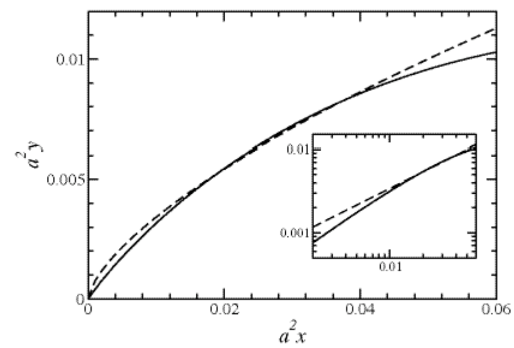

Fig. 1: Shape of the extension of a branched crack, in adequate dimensionless units. The dashed curve corresponds to a power law curve $y=x^{2/3}$. The inset shows the same plot in Logarithmic scale.

Using these exact results, the branching of a single propagating crack under mode I loading is also considered. It is shown that the formulation of the corresponding model problem is identical to the anti-plane case. The difficulty for solving completely the branching problem under plane loading configurations lies in the existence of two characteristic wave speeds and in the vectorial nature of the displacement field. However, this analogy allows the reasonable hypothesis that under plane loading configurations, the jump in the energy release rate due to branching is also maximized when the branches start to propagate quasi-statically. Using the Eshelby's approach, the branching of a single propagating crack under mode I loading is found to be energetically possible when its

speed exceeds a threshold value. Moreover, it is shown that an increasing fracture energy with the velocity results in a decrease in the critical velocity at which branching is energetically possible.
Finally, Using the asymptotic expansion of the stress field ahead a curved extension of a crack [18,19], the experimentally observed shape of the branches is recovered without introducing any additional parameter (see Fig. 1). It is shown that the length scale introduced by the curved extension of the branches is the geometrical length scale of the experiment. The quantitative comparison with the experimental results agrees with the theoretical findings [20]. The present study confirms that the continuum theory of brittle fracture mechanics provides the minimal ingredients for predicting the branching instability threshold, the branching angle and the subsequent paths of the branches.

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