

# PLASTICITY EFFECTS ON FATIGUE CRACK GROWTH AT HOLES

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## ABSTRACT

Severe loads can cause local plasticity to occur at stress concentrations, which are also common sites for fatigue crack formation. A coordinated analytical-experimental study was conducted to investigate the effects of this plasticity on fatigue crack growth behavior. A new cyclic shakedown methodology was developed to model the stress relaxation and redistribution caused by local yielding. In order to address the resulting complex stress gradients, new weight function stress intensity factor solutions for cracks at holes were derived from original boundary element analyses. The stress field changes can also change the local  $R$ -ratio, and the resulting effects on crack growth rates were modeled with crack closure concepts. When cyclic loading is so severe that cyclic yielding occurs at the edge of the stress concentration, the linear elastic stress intensity factor is no longer an adequate representation of the crack driving force, and an elastic-plastic crack parameter such as the range of the  $J$ -integral is required. Simple analytical techniques to estimate  $\Delta J$  were developed and validated against detailed elastic-plastic finite element models. A critical set of fatigue crack growth experiments were performed with Al 2124-T851. Conventional crack growth analysis methods were generally unsuccessful at predicting the test lives, but a comprehensive engineering methodology combining the new shakedown, weight function stress intensity factor, crack closure, and  $\Delta J$  models was successful.

## 1 INTRODUCTION

Fatigue cracks are prone to form at stress concentrations where cyclic stresses and strains are severe. Conventional fatigue crack growth (FCG) life prediction models are based on linear analysis methods (linear elastic stress fields, stress intensity factors (SIFs), etc.) that may be invalid when severe loads cause substantial nonlinear plastic deformation at fatigue-critical locations. Alternative engineering methods for nonlinear, elastic-plastic FCG behavior must address several distinct but related issues: estimating the elastic-plastic stress distribution in the uncracked body due to yielding, calculating the SIF for an arbitrary stress distribution, characterizing the driving force for FCG when linear elastic fracture mechanics is invalid, and addressing changes in the FCG rate due to plasticity-induced crack closure. In this paper, these four challenges are addressed with practical engineering models. Critical FCG experiments performed with Al 2124-T851 were used to evaluate the performance of the integrated engineering methodology.

## 2 CYCLIC SHAKEDOWN MODEL

Local plasticity (even in the absence of a crack) changes local stress and strain distributions. The resulting elastic-plastic fields can be determined by elastic-plastic finite element analysis, but this is not practical for routine engineering analysis. Simplified analytical methods have been derived to estimate the elastic-plastic fields from the known prior elastic fields. This methodology is sometimes called “shakedown” because reversed plastic deformation does not occur and the structure “shakes down” to a purely elastic response. The model discussed here accommodates reversed and repeated deformation and hence is called “cyclic shakedown.” Required data are structural dimensions, the uniaxial stress-strain curve in Ramberg-Osgood form, and all six stress components from a linear elastic stress analysis.

Two major tasks are performed in the cyclic shakedown methodology. The first task determines the stress relaxation at each nodal position from the linear elastic stress state. This “point relaxation” task incorporates a material hardening rule for isotropic materials proposed by Mroz [1] and Garud [2] with an incremental Neuber formulation [3] for non-proportional loading. The second task performs the load shedding that occurs due to point relaxation and re-distributes the surplus incremental forces and moments over the load bearing area. The current cyclic shakedown model addresses univariant stress distributions, but an enhanced model that addresses bivariant stress distributions is under development.

The cyclic shakedown methodology is illustrated in Figure 1, which shows the linear elastic and elastic-plastic stress distributions adjacent to a hole of radius  $\rho$  in a plate subjected to severe stressing (maximum applied stress 30 ksi, nominal cyclic yield strength 51 ksi, proportional limit near 41 ksi,  $K_t = 3.145$ ,  $R = 0.1$ ). In this example, only limited reversed deformation occurs, but a non-zero residual stress distribution remains upon unloading to zero load.

### 3 WEIGHT FUNCTION STRESS INTENSITY FACTOR SOLUTIONS

Most SIF solutions for cracks at holes are based on uniform remote tensile or bend loading, and therefore assume that the stress field near the hole follows the customary elastic distribution. However, when local stresses are perturbed by local yielding, alternative SIF solutions are required that admit arbitrary stress distributions on the crack plane. This requirement is met by the weight function (WF) method, which calculates the SIF by integrating over the crack surface the product of the stress in the uncracked body with the WF for the cracked structure.

New univariant WF SIF solutions for surface, corner, and through cracks at holes were developed [4]. The surface and corner crack solutions employed the approximate WF of Glinka [5-7] and a large matrix of highly accurate reference solutions for uniform and linear crack face tractions on cracks of various shapes and sizes in finite bodies. Reference solutions were generated using the FADD3D boundary element (BE) code [8]. The resulting WF solutions were verified using independent FADD3D analyses for stress gradients, as well as comparisons with finite element (FE) and literature solutions for uniform loading. The WF for the through crack was analytically derived from two-dimensional BE SIF solutions employed in NASGRO [9] and FE solutions for the stress distributions in the corresponding finite bodies. A new family of WF SIF solutions for cracks in finite bodies with bivariant stress fields is under development [10].

### 4 ESTIMATING THE RANGE OF THE J-INTEGRAL

If the cyclic deformation at the fatigue critical location is particularly severe, then the range of the linear elastic SIF will be an inadequate measure of the crack driving force. In this case, an elastic-plastic representation of the driving force, such as the range of the  $J$ -integral ( $\Delta J$ ), may be required. The total  $\Delta J$  solution is estimated as the sum of a fully plastic term and a first-order plastically (FOP) corrected elastic term. The fully plastic term,  $\Delta J_p$ , is usually negligible for cracks at holes, because the applied load is usually much lower than the global yield load of the cracked body. The FOP-corrected elastic term is defined as  $\Delta J_e = [\Delta K(a_e)]^2/E'$ , where  $\Delta K(a_e)$  is the linear elastic SIF range for an “effective” crack length  $a_e = a + \phi r_p(a)$ , and  $E' = E$  for plane stress and  $E/(1-\nu^2)$  for plane strain. Here  $r_p(a)$  is the plastic zone radius at the crack tip and  $\phi = 1/[1+(P/P_0)^2]$ , where  $P$  is the applied load and  $P_0$  is the plastic collapse limit load. Note that the FOP-corrected SIF range is calculated from the original elastic stress distribution in the uncracked body, not the elastic-plastic stress distribution determined by the shakedown model. The elastic-plastic stresses are used to calculate the maximum and minimum values of  $K$  in order to determine the local SIF ratio ( $R = K_{\min}/K_{\max}$ ), which is employed in the closure analysis (discussed below).

This  $J$  estimation scheme was evaluated for quarter-elliptical corner cracks at holes by comparisons against full numerical (elastic-plastic finite element) solutions. The numerical formulation was first validated by comparison against the WF SIF solutions for linear elastic conditions. A comparison of the elastic-plastic  $J$  estimate against the elastic-plastic FE solutions is shown in Figure 2 for three crack aspect ratios, eight crack sizes, three remote loads, and two material models. Different symbols indicate the opposite intersections of the crack with a free surface: the  $a$ -tip is in the bore of the hole, and the  $c$ -tip is at the plate surface. The agreement is strong, and any errors in the estimation scheme are conservative.

## 5 CRACK CLOSURE ANALYSIS

Fatigue crack growth rates are influenced by the deformation in the immediate vicinity of the crack due to the current and prior crack tip history. This near-tip deformation affects the opening and closing of the crack and thereby influences the fraction of the stress cycle that is actually effective in growing the crack. These plasticity-induced crack closure effects are most pronounced for cracks growing under severe loads, at different stress ratios, near stress concentrations, and under variable amplitude spectrum loading. Engineering tools such as FASTRAN [11] and STRIPY [12], both based on strip-yield models, have been previously developed to characterize this closure behavior.

The strip-yield model in NASGRO was integrated with the shakedown, stress intensity factor, and J-integral modules described earlier. Figure 3 illustrates the closure behavior, characterized by  $U = (S_{\max} - S_{\text{open}}) / (S_{\max} - S_{\min})$ , for a constant amplitude history. In this example, yielding at the edge of the hole caused a change in the local  $R$ -ratio, and this change was different at the  $a$ -tip than the  $c$ -tip. This change in local  $R$  resulted in changes in crack opening behavior as the crack grew, again different at the two opposite crack tips. The crack opening stresses were also inherently influenced by the severity of the maximum applied stress, expressed through the stress intensity factor.

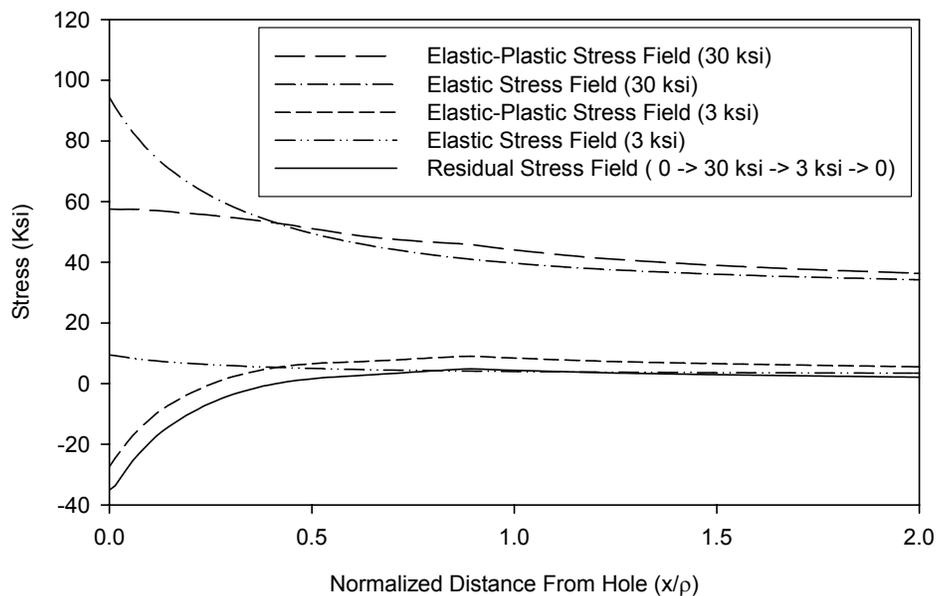


Figure 1. Elastic, elastic-plastic, and residual stress distributions near a hole for  $R = 0.1$  loading.

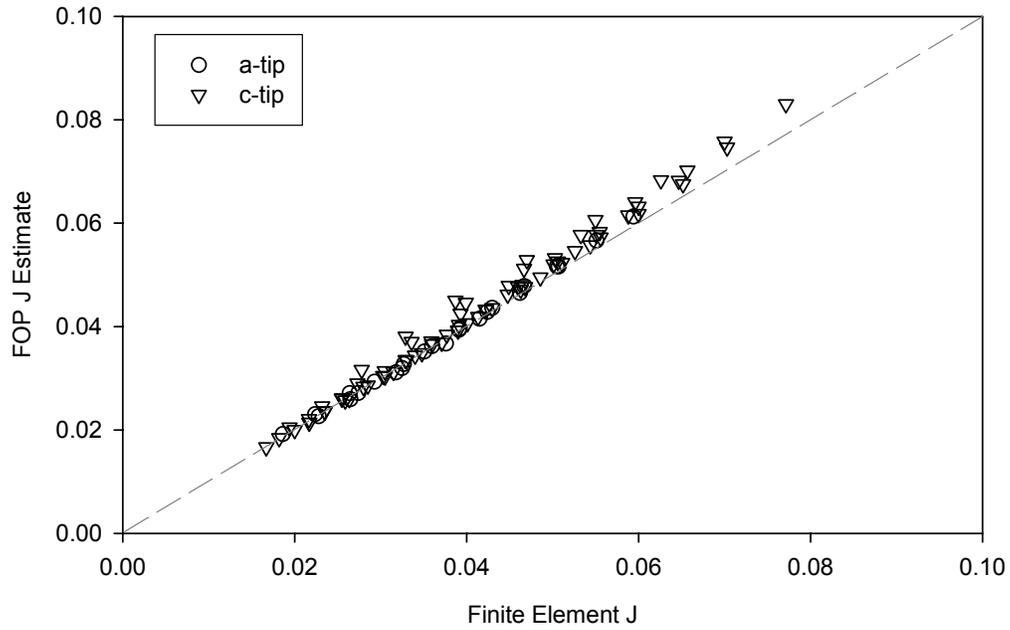


Figure 2. Comparison of FOP  $J$  estimate with FE  $J$  calculation for corner crack at hole.

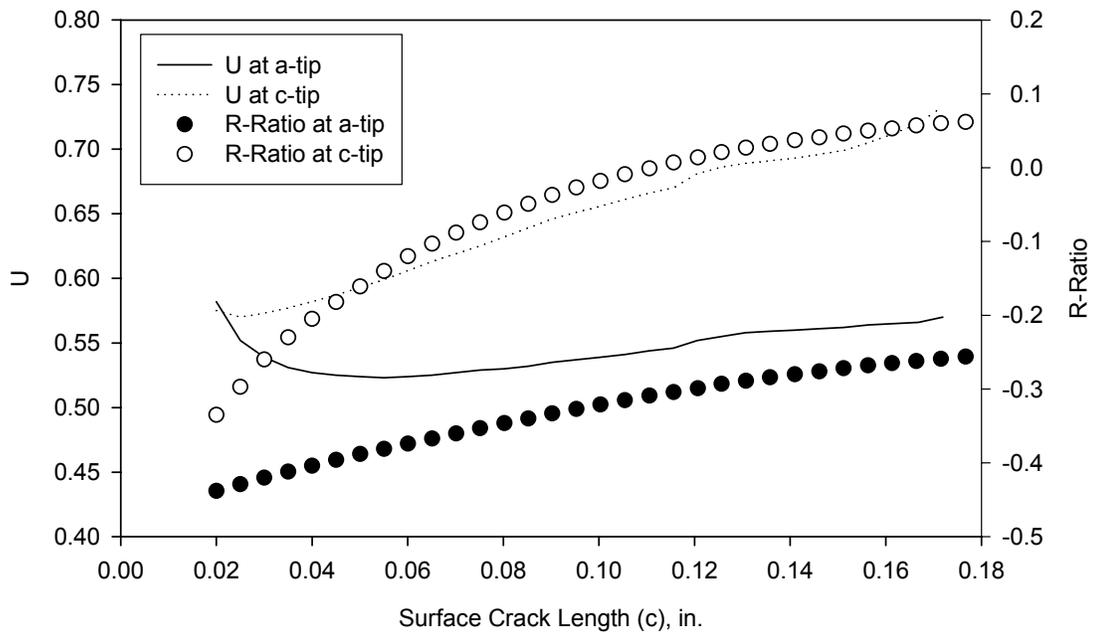


Figure 3. Local  $R$ -ratio and crack closure behavior for growing corner crack at hole.

## 6 EXPERIMENTAL EVALUATION

A critical set of FCG experiments was performed with aluminum alloy 2124-T851 under constant amplitude loading at three different maximum stresses (10, 20, 30 ksi) and two different stress ratios (0.1, -1) on a test coupon with a small initial corner crack at a hole. Test conditions ranged from no yielding at the edge of the hole to severe reversed yielding at the edge of the hole. Baseline FCG properties were generated at three stress ratios (-1, 0.1, and 0.5) on C(T) and M(T) specimens.

Both conventional FCG methods and the new models described above were used to predict the coupon results from the baseline properties, with no additional free parameters to adjust. The conventional models included FASTRAN 3.8, NASGRO 4.02 (strip yield model), and AFGROW 4.0007.12.11 [13]. The conventional models varied in their accuracy for the tests without yielding, but were generally unsuccessful at predicting FCG lives for tests with significant yielding. The new elastic-plastic models were generally successful at predicting FCG lives under all conditions considered. Figure 4 compares test data with various predictions for representative tests at a maximum nominal stress of 30 ksi (the most severe loading considered) and two different stress ratios. At an applied stress ratio of 0.1, yielding occurs at maximum load, but negligible reversed yielding occurs in the uncracked body, and so the structure shakes down to nominally elastic behavior. At an applied stress ratio of -1, cyclic yielding occurs in the uncracked body on every load reversal. The conventional models underpredict the life at  $R = 0.1$ , because they do not account for the shakedown relaxation of stress at the hole. The conventional models overpredict the life at  $R = -1$ , because they do not account for the enhanced elastic-plastic driving force due to cyclic plasticity. The new model addresses both of these conditions.

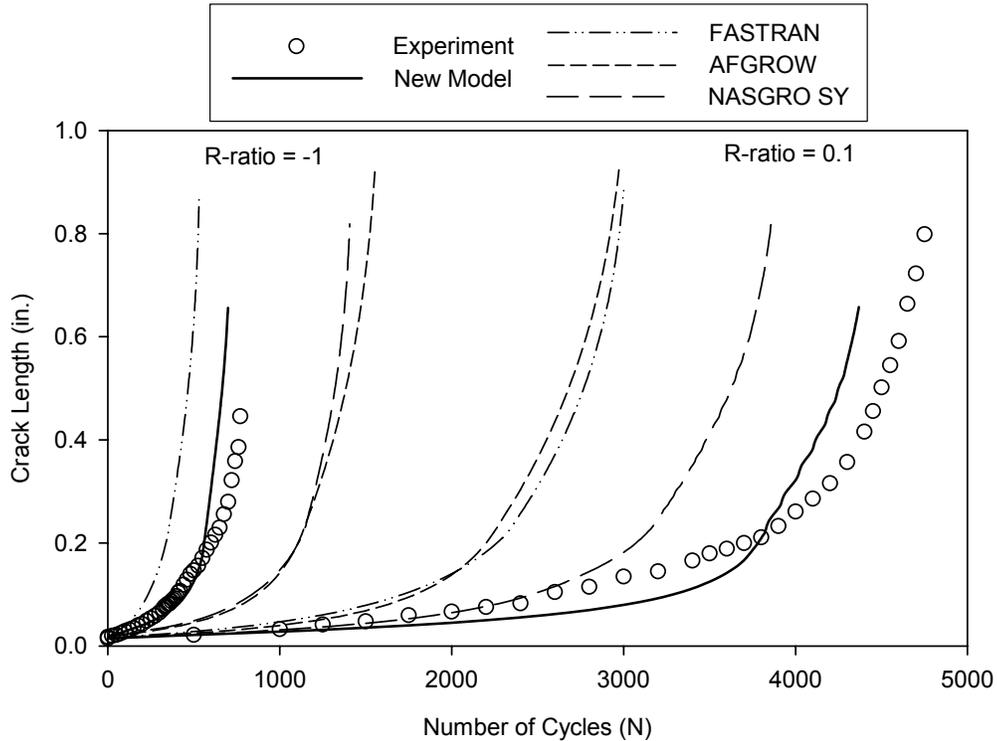


Figure 4. Comparison of experimental FCG data with conventional and new models.

## 7 SUMMARY AND CONCLUSIONS

Plasticity can have numerous effects on fatigue crack growth at holes, in some cases accelerating growth rates, and in other cases retarding growth rates. Conventional crack growth models, which generally do not treat these plasticity effects, are sometimes unable to predict FCG rates accurately. A new engineering methodology for plasticity effects has been developed, and initial evaluation indicates that it is generally successful in predicting FCG rates. The new methodology is being incrementally implemented in the NASGRO computer code for future distribution.

## 8 ACKNOWLEDGEMENTS

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