IDENTIFICATION OF EXTERNAL LOADS TO DYNAMIC SYSTEMS AS THE METHOD OF DEFECTS DEFINITION

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ABSTRACT

The problem of defects definition of the real object is reduced to the problem of identification of additional external load z to some subsystem of initial system which describes the motion of real object [1, 2]. This additional external load z is related with the mathematical description of inspected defect (parameter) of real object. The problem of identification z with using experimental measuring of initial external loads and response x of subsystem is considered here. Tikhonov's regularization method is used for solution of ill-posed (instable) problem of additional external load identification [3, 4]. Here the method of choice of special mathematical model was used for improvement of accuracy of regularized solution [5, 6, 7]. If the inspected defect is absent then $z = 0$. The change of the inspected defect in process of work will lead to the change of additional external load so that the motion of real object continues to be described by the old incorrect mathematical model. The deviation of the inspected defect from norm might be estimated on the basis of z change. The main advantages of such approach are:

i) there is no need to use any testing signals;

ii) the possibility of the continuous observation of the inspected defect (parameter);

iii) the possibility to use the simple mathematical models.

As an example the rapid evaluation of rotor unbalance characteristics was considered in different statements [8]: early evaluation, the most plausible evaluation, the guaranteed evaluation.

1 INTRODUCTION

It is assumed that the motion of real object without defects is being described by some mathematical model. The part of variables of state $x_j(t)$ $(1 \leq j \leq n)$ of studied mathematical model permit the direct experimental measuring. The function $x_j(t)$ obtained experimentally can be interpreted as two external loads $C_j\, x_j(t)$ and $- C_j\, x_j(t)$ (C_j – const) to the initial system. Then the initial system can be reduced to the more simple equivalent system (or systems). Such transformation is be called the "j-section" [2]. The subsystem of the initial system with all known initial external loads (except additional load) and one the known variable of state $x_1\,(t)$ are being picked out with the help of the series of "sections" of such kind. This subsystem has to contain the inspected physical defect the right size of which is taken into consideration within the mathematical description. The additional external load $z(t)$ is introduced into the mathematical description of the inspected defect. Then the problem of identification of external load $z(t)$ by results of experimental measuring of external loads to subsystem and response $x_1(t)$ is being investigated. If the inspected defect has a right value then $z(t) = 0$. The change of inspected defect in process of work will lead to change of additional external load so that the process into a real object continues to be described by the old incorrect mathematical model. The deviation into the inspected defect can be judged by character of change $z(t)$.

The linear integral equation of first kind for unknown function $z(t)$ can be obtained in many cases if the use of the simplest mathematical model of motion

$$A_p z = u_\delta, \qquad (1)$$

where A_p is a linear integral operator $(A_p\colon Z \rightarrow U)$; Z, U are the B functional spaces.

In this case an inverse operator A^{-1}_p is not defined for all $u_\delta \in U$ and is not continuous on U [3].

It is supposed that operator A_p depends continuous on vector of parameters of mathematical model $\boldsymbol{p} = (p_1, p_2, \dots p_m)^*$, $\boldsymbol{p} \in R^m$. This parameters are determined inexact as a rule with some error and by virtue of it they can accept values in known limits $p_i^0 \le p_i \le p^1{}_i$, $i = 1,2,3, \dots,m$. Therefore, the vector of parameters $\boldsymbol{p}$ can not be defined precisely and that it can accept values in some closed area $\boldsymbol{p} \in \boldsymbol{D} \subset R^m$ [8]. A certain operator A_p in (1) corresponds to each vector $\boldsymbol{p} \in \boldsymbol{D}$. Operators A_p form some class of operators $K_A = \{A_p\}$. The function u_δ is obtained from experiment with a known error δ: $\| u_T - u_\delta \|_U \le \delta$ (u_T is an exact response of system on exact external impact).

Let us denote by $Q_{\delta p}$ the set of functions which satisfy the equation (1) with the exactness of experimental measurements and with fixed operator A_p:

$$Q_{\delta,p} = \{ z : z \in Z,\ \| A_p z - u_\delta \|_U \le \delta \}. \tag{2}$$

The set of $Q_{\delta,p}$ is unbounded set in the norm of space U [3, 4].

It is assumed that the operators A_p are linear. Let us designate through h size of the maximal deviation of the operators A_p from K_A:

$$\sup_{p_1;\, p_2 \in \boldsymbol{D}} \| A_{p_1} - A_{p_2} \|_{Z \to U} \le h.$$

Exact operator A_T has the structure as the structure of A_p. Parameters of operator A_T belong to domain $\boldsymbol{D}$ also. The set of possible solutions has the form in this case (problem of recognition):

$$Q_{h;\delta} = \{ z : z \in Z,\ \| A_p z - u_\delta \|_U \le \delta + h \| z \|_Z \}.$$

The set of $Q_{h;\delta}$ is unbounded set in norm of space Z as A_p is compact operator [3].

2 PROBLEMS STATEMENT AND METHODS SOLUTION

The method of Tikhonov regularization for equation with inexact given operator is possible way to obtain the stable solution of problem (1) [3, 4]. Let $\Omega[z]$ is the stabilized functional which is defined on Z_1 (Z_1 is everywhere dense set in Z). Let us denote by z_0 the regularized solution of equation (1):

$$\Omega[z_0] = \inf_{z \in Q_{h;\delta} \cap Z_1} \Omega[z] \tag{3}$$

The problem (3) can be reduced to solution of following problem [5]

$$\Omega[z_0] = \inf_{A_p \in K_A} \inf_{z \in Q_{\delta;p} \cap Z_1} \Omega[z]. \tag{4}$$

This solution is possible to interpret as *lowest guaranteed evaluation* of the exact solution in sense of chosen stabilizing functional $\Omega[z]$. However such interpretation of the approximate solution has no sense in some inverse problems [9].

For example, it is necessary to consider the following problemin Krylov's inverse problem

$$\Omega[z_1] = \sup_{A_p \in K_A} \inf_{z \in Q_{\delta;p} \cap Z_1} \Omega[z]. \tag{5}$$

This solution is possible to interpret as *early evaluation* of the *additional external load*.

Let's consider the extreme problem:

$$\| A_p z^p - u_\delta \|_U \le \inf_{z_a} \sup_{A_c \in K_A} \| A_c z_a - u_\delta \|_U, \; \boldsymbol{a}, \boldsymbol{c} \in \boldsymbol{D} \text{ for all } A_p \in K_A. \quad (6)$$

Let us name function z^p as *the most plausible evaluation of exact additional external load.*
In work the questions of existence of the solution of the specified problem and its stability to small changes of the initial data are considered. The algorithm of the approximate solution is offered [10, 11].
The suggested approach is illustrated by the help of the problem ofrotor unbalance identification [1].

3 THE IDENTIFICATION OF THE UNBALANCE CHARACTERISTICS OF A ROTOR

Current methods of unbalance definition of machine rotor in their own bearings are not effective in case when the unbalance arises in the machine work process as they demand the special conditions of work or installation of trial plummets [12, 13]. Besides, these methods do not give the complete information about the location of unbalance if the rotor has a large size along the axis of rotation. The suggested algorithm of unbalance evaluation uses the experimental data about accelerations of rotor supports in two mutually perpendicular directions during the work for few rotor rotations as the initial information. This algorithm doesn't demand the special conditions of work or installation of trial plummets.
Let us consider a flexible rotor rotating on two non-rigid supports. The movement of rotor is described by the system ordinary differential equations of the 18th order. The projections of unbalance force and of unbalance moment z_1, z_2, z_3, z_4 have been chosen as the additional external loads to mathematical model of motion. If the unbalance is absent then the functions z_1, z_2, z_3, z_4 will be equal to zero.
The equations for the unknown functions z_1, z_2, z_3, z_4 are analogous to (1).
It is supposed that the vibration support have been recorded with the help of acceleration transducers. These vibrations define the right-hand side of equation (1) [2].
For examination of suggested algorithms of unbalance characteristics evaluations there was calculated the case when vibrations of support are the results of mathematical simulation of rotor vibrations with given unbalance. The size of rotor unbalance were chosen as: m_r = 0.5kg by r = 0.25M, h = 0.25M, ϑ = 0.5rad (r is the radius of rotor, m_r is the mass of unbalance reducing to a surface of rotor, h is unbalance arm, ϑ is angular deviation of the factor of unbalance with respect to correction plane).
The sizes of initial data inaccuracy were chosen as the following:

$$\| u_T - u_\delta \|_U \le \delta = 0.08, \qquad \sup_{p_1;\, p_2 \in \boldsymbol{D}} \| A_{p_1} - A_{p_2} \|_{Z \to U} \le h = 0.12.$$

The results of identification of *the most plausible evaluation* of unbalance are the following: m_r= *0.42kg* by *r = 0.25M, h = 0.22M, ϑ = 0.47rad*. The parameter of regularization α are defined by method the discrepancy [3]. The *lowest guaranteed evaluation of unbalance* has size: m_r= *0.20kg* by *r = 0.25M, h = 0.12M, ϑ = 0.21rad*. Evaluation of unbalance as early evaluation has the following size: m_r= *0.42kg* by *r = 0.25M, h = 0.21M, ϑ = 0.46rad.*

This method permits rapidly to evaluate all characteristics of unbalance on working machinery in real time. It can be used for technical diagnostics of unbalance and for balancing of rotors in their own bearings. The method can be adapted at when measuring velocity or displacement of supports.

The suggested approach to problem of defects definition can be used in wide class of problems as method of preliminary evaluation of defects.

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