OSCILLATING FRACTURE PATHS IN THIN BRITTLE SHEETS: WHEN GEOMETRY RULES CRACK PROPAGATION

B.AUDOLY[1], P.M.REIS[2], B.ROMAN[3]
[1]Laboratoire de Modélisation en Mécanique, UMR 7607 CNRS/UPMC, 4 place Jussieu, case 162, 75005, Paris, France
[2]Manchester Centre for Nonlinear Dynamics, Dept. physics and astronomy, University of Manchester, M139PL UK
[3]Physique et Modélisation des Systèmes Hétérogènes, UMR 7636 CNRS/ESPCI, 1O rue Vauquelin, 75005 Paris, France

ABSTRACT

We report a new kind of quasi-static oscillatory crack propagation when a cutting tool, of moderately large width, is driven through a thin brittle polymer film (Roman [1], Ghatak [2]). In our experiments, a cutting tool is perpendicularly driven through a brittle thin sheet, clamped at its boundaries, and progressively cuts the material as it advances. For large enough tools, the crack tip follows a highly periodic path, leaving behind a striking oscillatory pattern. The amplitude and wavelength of the oscillatory crack paths scale linearly with the width of the cutting tip, over a wide range of length scales, but are independent of both the width of the sheet and the cutting speed. We propose a geometrical model (Audoly [3]) that accurately reproduces the behaviour observed experimentally, far away from threshold. The central idea of our formulation is a coupling between the Griffith criterion for crack propagation and out-plane-deformations of the film. We show that, for our case, the propagation of cracks in brittle sheets follows a simple geometrical construction.

1. INTRODUCTION

A fascinating and yet unsolved problem in fracture theory concerns the direction of propagation of the crack tip, and its associated instabilities. This can be stated as follows: when a piece of glass breaks, can the shape of the resulting pieces be predicted? Recently, well-controlled experiments have yielded a wealth of interesting behaviour that poses a challenge to existing theoretical formulations. An oscillatory instability in dynamic cracks has been observed when a pre-tensioned thin rubber sheet is pierced (Deegan [4]) but the nature of the underlying mechanism is still unclear. Another example is the controllable quasi-static propagation of oscillatory cracks in a thin strip of glass or silicon submitted to a thermal field (Yuse [5], Ronsin [6] Deegan [7]), which, despite its apparent simplicity, has been stimulating many theoretical studies (Adda-Bedia [8], Yang [9]). We have recently reported results on oscillatory fracture paths in a new experimental context: an object, which we denote by *cutting tool*, is driven through a brittle thin polymer film, clamped at its boundaries, and progressively cuts the material as it advances. The tool is oriented perpendicularly to the film surface and driven parallel to the film length, see Fig. 1. For large enough tools, the crack tip follows a highly reproducible periodic path, leaving behind a striking oscillatory pattern that spans a wide range of length scales, as shown in the two examples

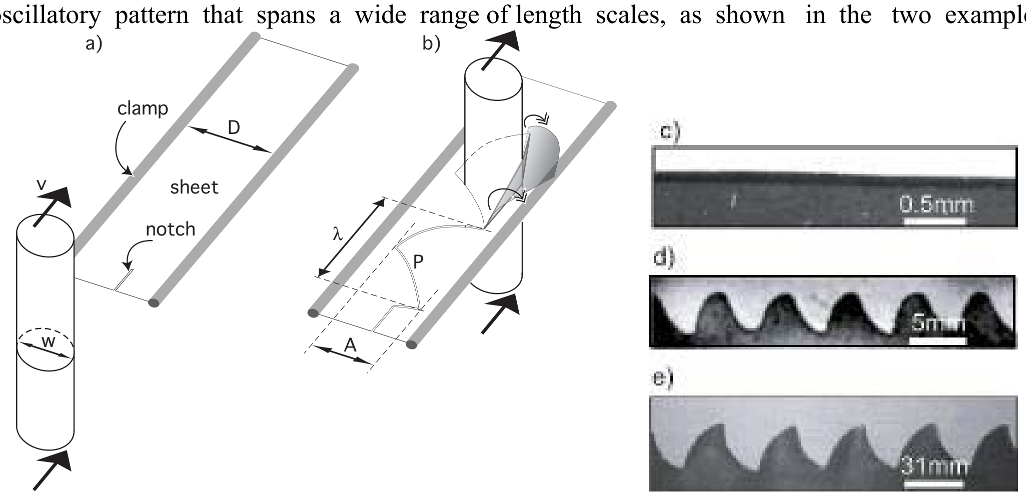

Figure 1 : *a) and b) Schematic diagram of the experimental set up. A cylindrical cutting tip is forced into a clamped thin polymer sheet with a notch, leading to an oscillatory crack path P. a) Initial configuration of the experiment. b) Typical configuration during fracture. The advance of the cutting tip through the sheet leads to out of plane deformations (double arrows and region in grey). c-e) Painted edge of the sheet (dark region) seen from above after the two halves of the film have been taken apart (only one side is shown) and rescaled (polypropylene 27µm thick) for three cutting tip widths: c) w=0.15mm (straight path), d) w=5mm (oscillatory path), e) w=31mm (oscillatory path).*

presented in Fig.1(d) and (e). In fact, even doing the experiment by hand yields surprisingly regular patterns which indicates the robustness of the phenomena. In this communication, we first recall these recent experimental results, and then present a reductionistic geometrical model that accurately reproduces the behaviour observed experimentally, far away from threshold. These ideas should have practical applications since the precise cutting of brittle thin sheets is common in manufacturing and the materials we study are routinely used in the packaging industry.

2. EXPERIMENTAL SET-UP AND RESULTS

A schematic diagram of our apparatus is presented in Fig.1(a). A thin flat sheet is clamped along its lateral boundaries and mounted on a linear translation stage. This stage is driven at constant speed, v, towards a fixed object, the *cutting tool*, which had either rectangular or circular cross-section, with a variety of widths (0.05mm<w<60mm). A camera was mounted directly above the apparatus such that the propagating crack was imaged in the cutting tool's frame of reference. The sheet was firstly prepared with a notch, on one of its side boundaries, in order to initiate the crack in a well defined position. Both bi-oriented polypropylene and cellulose acetate thin sheets were investigated, with thicknesses ranging from 25 to 130μm (sheet's Young's modulus is Y=1-2 GPa and fracture energy Γ=2-5kJ/m^2). Although polymeric, these materials are brittle since they have been severely stretched when processed into thin sheets: they undergo minimal plastic deformation during fracture propagation but, being thin, can sustain large bending without crack initiation. This explains why they are widely used in the packaging industry (resistant but easy to tear once a notch is started). The oscillatory paths discussed below were not observed in ductile materials. The detailed description of our experimental set-up has been presented elsewhere (Roman [1]).

As the thin sheet is forced through the fixed object, the material is cut, leaving behind a well defined path. For large enough cutting tips, the resulting path is oscillatory, two examples of which, for significantly different sizes of the cutting tip, are shown in Fig. 1(d) and (e). In this oscillatory regime, the non-sinusoidal oscillatory paths resemble a series of shark blades; the fracture path is made up of smooth curves connected by sharp kinks. Fast dynamic propagation occurs immediately after the change in direction (kink) of the crack path which then gives way to a longer regime of quasi-static propagation. As one would expect, the path left behind a thin cutting tool is straight, Fig 1(c). Our results point to a new instability in the fracture of thin polymer films from straight to oscillatory patterns, as the size of the cutting tip is increased.

In this communication, we focus primarily on the regime well above threshold. The amplitude A and wavelength λ of the oscillatory paths, see Fig. 1(b) for definition, depend linearly on the width of the cutting tool, w. We found that this oscillatory behaviour is independent of the material and thickness of the sheet, its width D, and the cutting speed, within the ranges we have explored (Roman [1], Ghatak [2]). These remarkable features are consistent with the robustness of the phenomenon (one can obtain very regular pattern driving a key in a plastic package), but are rather surprising: far above threshold, the only relevant parameter in the problem is the size of the cutting tool.

Ghatak [2] suggested that the crack path can be constructed by replicating arches of prolate cycloids. The stacking of two consecutive arches, the second being mirrored about the pattern's central axis, yields a single period of the oscillation. Even though this approach describes the experimentally observed crack paths to within 15-20%, the two fitting parameters are entirely *ad hoc* and the formulation is based on a series of unphysical assumptions. In this paper we propose a physical model for crack propagation in our system, based on a coupling of the Griffith criterion for crack propagation and out-of-plane deformations. Our simulations exhibit excellent agreement with the experimentally observed phenomena and accurately account for both the shape of the crack paths and the existance of successive dynamic and quasi-static regimes of propagation of the crack tip.

3. A GEOMETRICAL MODEL FOR PROPAGATION

The problem of crack propagation in thin sheets with large out-of-plane deformations is challenging as it involves a coupling between fracture theory and elasticity of thin plates. The latter is described by non-linear partial differential equations with a complex structure. Here we propose a reductionistic approach. According to Griffith's criterion (see Freund [10]), the crack propagates when the release rate of elastic energy overcomes the fracture energy Γ. The elastic energy that we consider is specific to thin sheets; it contains a bending term (associated with curvature) and a stretching term (associated with in-plane deformations), see Pogorelov [11]. For large out-of-plane deformation of thin sheets, bending energy is negligible compared to stretching energy, i.e. thin sheets are easy to bend. Since the dimensions of the cutting tool are significantly larger than the sheet's thickness (h/w~10^{-3}) we consider the limiting case of an infinitely thin sheet and take the reasonable assumption that the bending energy is zero.

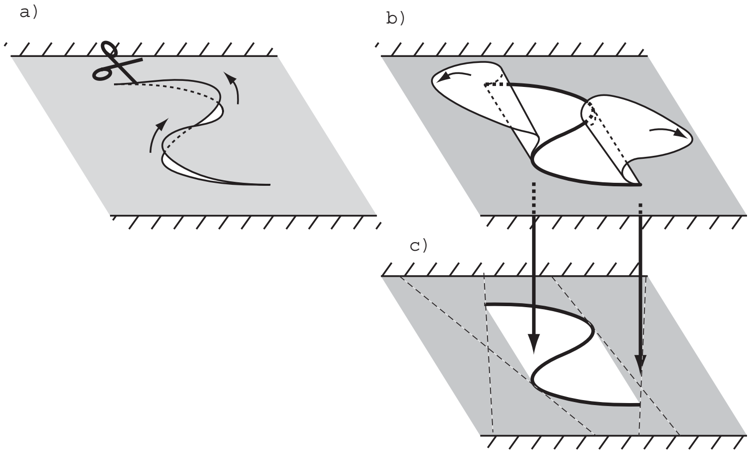

Figure 2: *For a given fracture path, (a), in a thin sheet clamped along its lateral edges, the "soft zone" is defined as the area that can be deformed by pure bending, (b), that is with a negligible elastic energy. This soft zone, shown in white is the* convex hull *of the crack path. (c) The model operates on the two dimensional projection of the strip rather than on the full three dimensional configuration.*

Consider an arbitrary crack path in the sheet (Fig. 2a). The convex hull of the crack path defines the *soft zone*, shown in white in Fig. 2(b&c). This zone has the remarkable mechanical property that it can undergo large out-of-plane deformation by pure bending without stretching. As long as the projection of the cutting tool is fully contained in this soft zone, the sheet merely bends away to accommodate its presence (Fig. 3a). The resulting elastic energy is pure bending and can be neglected, being too small to make the crack propagate.

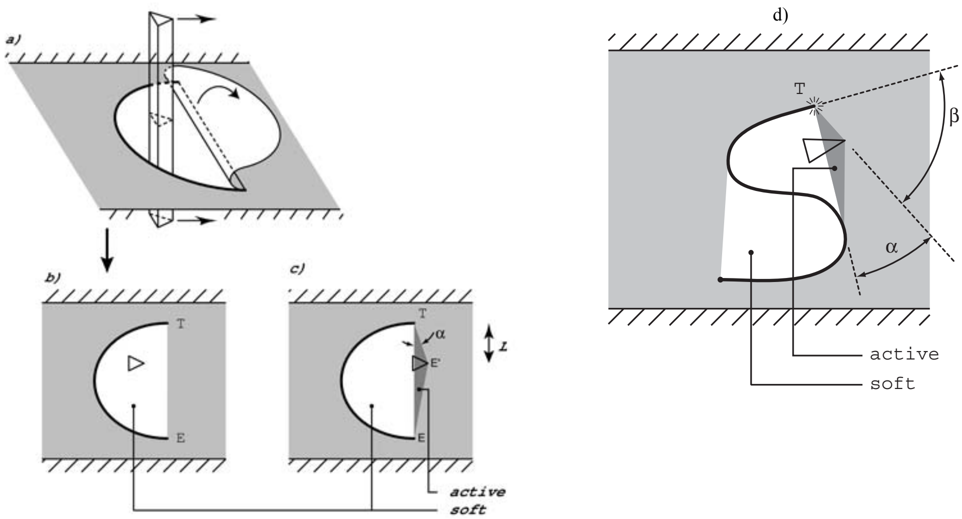

Figure 3: *When the cutting tip, here with a triangular cross-section, is moved from within the soft zone — (a), three dimensional view, (b) 2D projection — into the hard zone (c), the sheet undergoes stretching: the active zone, shown in dark grey is repelled by the tool tip to the right, and the line TE is strecthed into TE'E. This is the basis of our estimate of the elastic energy in the field which, along with Griffith's criterion, leads to the propagation criterion in Eq. (1). The local direction of propagation is given by an angle β.*

In contrast, when the projection of the cutting tool moves beyond this soft zone, the film is stretched and stores a substantial amount of elastic energy. Let us call *hard zone* the part of the film that is not in the soft zone, shown in grey in Fig. 2c. Any point in this hard zone belongs to many uninterrupted line segments drawn on the sheet that extend to the clamping frame on both sides (dashed lines in Fig. 2c) : when the hard zone is perturbed by the presence of the tool, these segments, whose endpoints are fixed, must stretch. This leads to in-plane tensile stresses in the film which can eventually trigger crack propagation.

Let us introduce a third zone, the *active zone*, which is the part of the hard zone that is perturbed by the penetration of the tool. From Fig. 3d, the active zone can be computed as the set difference of the convex hull of the crack path union the projection of the tool, minus the soft zone (earlier defined as the convex hull of the crack path alone). This yields the triangle in dark grey in Fig. 3(c&d). To estimate the stretching energy of the film, we note that the typical tensile strain in this region is given by the strecthing of the line TE in the reference configuration, which becomes the broken line TE'E in the presence of the tool. This yields an estimate of the stretching energy as $E \sim Ye^2v$, where Y is the Young's modulus of the sheet, $e \sim \alpha^2$ (for $\alpha << 1$) is the strain of the sheet, α is the angle at the vertex of the active zone near the crack tip (Fig. 2c,d) and v is the volume of material strained (surface of active zone times thickness). Following Griffith's approach, we balance the release rate of elastic energy stored in the sheet with the dissipation of energy during crack propagation. The corresponding criterion for crack propagation can be stated as follows:

no propagation if $\alpha < \alpha_c$

propagation when $\alpha = \alpha_c$ *with* $\alpha_c \sim [\Gamma/(YL)]^{1/4}$, (1)

where α_c, is a critical angle above which propagation of the crack tip occurs, L is a typical lengthscale (distance from the cutting tool to the fracture tip, see Fig. 3c). This picture holds if $\alpha_c << 1$, hence $\alpha << 1$, which is indeed the case for the typical values of Y, Γ and L measured experimentally.

There remains to predict the direction of crack propagation. To do so, one generally uses the principle of local symmetry. This phenomenological criterion, which states that the cracks propagates preferentially in mode I, requires the computation of the stress intensity factors. Here we replace it by a simplified version, based on the physical remark that the stresses near crack tip arise mainly due to stretching of the line TE. As a result, the direction of propagation should be close to perpendicular to the edge of soft zone (TE). We will thus assume that direction of propagation is given by a fixed angle β with respect to the line joining cutting tip-crack tip, see Fig. 3d, both β and $\beta+\alpha$ being close to $\pi/2$.

Using the specific mechanics of thin sheets and Griffith's criterion, the propagation of a crack can thus be computed through the following *two dimensional geometrical construction*: at each time step, compute both the convex hull of the crack path (soft zone) and the convex hull of the crack path union the tool projection on the other hand. Then, do the set difference between this two sets (yielding the active zone), and measure the angle α at the tip of the active zone. If $\alpha \geq \alpha_c$, propagate the crack in the direction β, until $\alpha < \alpha_c$. Then, update the indenter position and loop. This procedure assumes that the crack reacts instantaneously to the indenter, taking the dynamic crack velocity as infinite. This is reasonable, given the separation of scales between sound speed in the material and speed of the cutting tool which is typically of the order of ~1mm/s.

4. COMPARISON WITH EXPERIMENTS

Upon numerical integration, we found that this geometrical model generates oscillations for a wide range of parameters and initial conditions, with crack path fitting extremely well the experiments (See Fig. 4, for $\alpha_c = 0.29$ $\beta = \pi/2 - 0.13$, the two only adjustable parameters of the model). We point that the simple estimate of α_c by Eq. (1) yields the correct order of magnitude, $\alpha_c = O(0.1)$, and that β is close to $\pi/2$ as expected. Moreover, the model reproduces the temporal evolution of the crack, and features quasistatic phases of propagation followed by sudden change of direction and a dynamic jump to a new position, as in the experiments. These large jumps are clearly visible in the plots of the x and y positions of the crack tip as a function of the advance of the tool, shown in Fig. 4. Therefore, these dynamic jumps captured by our model are also observed in the experiments and arise from the coupling of crack propagation and the mechanics of thin plates (Audoly [3, 12]).

Moreover, our model accounts for the fact that the pattern is independent of the width of the sheet (Roman [1], Ghatak [2]): the geometrical construction has only one length scale, the size of the cutting tool. We found (Audoly[3]) that the pattern amplitude and wavelength depend only weakly on the parameters (α_c ,β). Since α_c is itself weakly dependent on the tool size L and the film mechanical properties (small exponent, ¼), our model predicts that the pattern should be almost proportional to the cutting tip size, independently of material properties, film thickness, film width, and velocity of the cutting tool. All these features were indeed observed experimentally (Roman [1]). Moreover, the ability to describe these complex fracture morphologies using a rather simple geometrical construction is consistent with the fact that the phenomenon is highly robust and can be observed even when perfoming the experiment at hand.

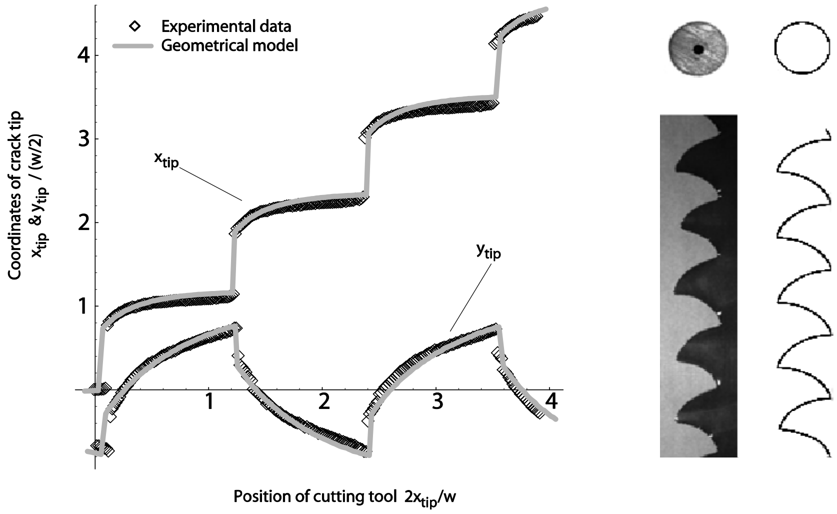

Figure 4: (*a) Time-series for the coordinates of crack tip (i.e. as a function of tool position since cutting speed is constant.* x_{tip}/y_{tip} *along/perpendicular to the direction of cutting. Experimental (black symbols) and model prediction (solid grey lines). (b) : Experimental (b) and numerical (c) crack morphologies, for the same cylindrical cutting tool (circles with 31mm diameter, on top).*

Beyond these scaling laws, our model accurately compares with the experiments and reveals the mechanism for the oscillation, which can be understood as follows. Because the soft zone does not contribute to the crack motion, the cutting tip only pushes on one side of the crack lip at a time. Suppose it pushes on the rigth lip; because the angle α_c is small and β is of the order of $\pi/2$, the crack will propagate even further to the right. The model explains why straight paths are unstable, far from threshold.

5. CONCLUSION

We have presented a model based on Griffith's criterion for the propagation of cracks in brittle thin sheets. We have shown that our model captures the mechanism of a new oscillatory crack path instability observed when a blunt cutting tool (with radius much larger than the sheet's thickness) is pushed through a clamped film. A remarkable feature of the model is that the crack propagates according to a *purely geometrical, two dimensional construction*. It has two angular parameters, $\alpha_c \ll 1$ and $\beta \sim \pi/2$, that embody the fracture and mechanical properties of the film. This construction was derived from the specific mechanical properties of thin plates, whose mechanical response is strongly connected with geometry. We stress the fact that the oscillatory nature of the crack, the existence of kinks, as well as the alternatively dynamical and quasistatic regimes of propagation were not *a priori* built into the model. In fact, this model provides a general framework for analyzing crack in brittle thin plates: it should be applicable to arbitrary, non-circular tool geometries, and also to completely different loading configurations.

[1] Roman B., Reis P.M., Audoly B., De Villiers S., Viguié V., Vallet D., *Oscillatory Fracture Paths in Thin Elastic Sheets,* C.R. Mecanique 331 (2003).
[2] Ghatak A. and Mahadevan L. *Crack Street: The Cycloidal Wake of a Cylinder Tearing through a Thin Sheet*, Phys. Rev. Let. 91, 215507 (2003)
[3] B. Audoly, P.M. Reis B. Roman, in preparation
[4] Deegan R., Petersan P., Marder M. and Swinney H., *Oscillating Fracture Paths in Rubber*, Phys. Rev. Let. **88** (2002).
[5] Yuse A. and Sano M., *Transition between Crack Patterns in Quenched Glass Plates*, Nature **362**(1993) 329
[6] Ronsin O., Heslot F. and Perrin B., *Experimental Study of Quasistatic Brittle Crack Propagation,* Phys. Rev. Let. **75**(1995).
[7] Deegan R., Chheda S., Patel L., Marder M., Swinney H. Kim, J.-h. and de Lozanne A., *Wavy and Rough Crack in Silicon*, Phys. Rev. E. **67**, 66209 (2003)
[8] Adda-Bedia M. and Pomeau Y., *Crack Instabilities of a Heated Glass Strip*, Phys. Rev. E **52** (1995) 4105
[9] Yang B. and Ravi-Chandar K., *Crack Path Instability in a Quenched Glass Plate*, J. Mech. Phys. Solids **49** (2001) 91.
[10] Freund L. *Dynamic Fracture Mechanics,* Cambridge university press 1990.
[11] Pogorelov A., *Bending of Surfaces and Stability of shells*, Translation of mathematical monographs, AMS, 1988.
[12] Audoly B., Reis PM, Roman B, http://www.lmm.jussieu.fr/platefracture/