FRACTURE TOUGHNESS OF SNOW: SIZE EFFECTS IN A QUASI-BRITTLE MATERIAL

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ABSTRACT

Dry snow slab avalanche release is preceded by a shear failure in a weak layer or an interface of the snowpack, followed by a fracture in tension. Both these processes can be described by fracture mechanics. For a better understanding of the slab release process, measurements of the relevant snow mechanical properties, such as the fracture toughness, are essential. Knowledge of these properties is necessary for a snow slope stability analysis that includes fracture propagation. The purpose of this work was to measure the snow fracture toughness in mode I (tension), and to determine whether or not snow fracture toughness is affected by a size effect for the used specimen size. The latter is limited because of practical difficulties in handling big specimens. Edge-cracked beam-shaped snow specimens cut from a homogeneous layer of naturally deposited snow were loaded in three point bending. All experiments were performed in a cold laboratory. In order to quantify the size effect, specimens of the same shape but of four different sizes were used. Preliminary results indicate that fracture toughness is affected by the specimen size. The reason for this behaviour is the size of the fracture process zone, which for snow is within the range of the ligament widths of the used specimens.. Actually, these specimens showed a non-linear, quasi-brittle behaviour. In order to be used in slope stability models, the measured fracture toughness values require a corresponding size correction.

1 INTRODUCTION

The natural snowpack is a layered material consisting of snow layers with different properties. If a layer of well consolidated snow (a slab) is poorly bonded to the underlying layer, a snow slab avalanche may form. Before the release of a dry snow slab avalanche several damage processes occur. Damage accumulation in a weak layer of the snowpack or at the interface between two snow layers leads to an initial failure in shear along the weak layer (Schweizer, et al. [1]). When the tensile forces in the snow slab overlaying the growing fractured zone exceeds the tensile strength, a fracture in tension occurs and the slab avalanche is released. Both fracture processes can be described and understood by the theory of fracture mechanics. Thus, the corresponding material characteristics such as the fracture toughness have to be known. They are essential to well-founded and reliable slab release models.

Recently, the first fracture mechanical experiments were performed to determine fracture toughness of snow (Kirchner, et al. [2]; Schweizer, et al. [3]; Kirchner, et al. [4]; [5]). They used cantilever beam tests for their experiments and determined fracture toughness by applying linear elastic fracture mechanics (LEFM). However, Schweizer et al. [3] pointed out that the standard size requirements were not fulfilled for the specimen size used (10 cm × 20 cm × 50 cm). Their values for snow fracture toughness in tension range from 0.1 to 1.5 kPa $m^{1/2}$ depending on snow density. Bazant et al. [6] related fracture toughness to the avalanche release process. They found that fracture toughness in shear is approximately proportional to snow thickness to the power of 1.8. Dempsey et al. ([7]; [8]) tested sea ice over a very large sample range from 0.1 m up to 100 m.

With Bazant's empirical size effect law (Bazant and Planas [9]), they were able to predict the fracture load for test sizes 27 times larger than the ones used to determine the size effect law.

In the present study, three-point bending test was used to measure snow fracture toughness. Unlike the cantilever beam test as previously used for snow, the crack loading mode is pure mode I. The aim was to compare the different tests and to preliminarily assess the size effect for the specimen size of our experiments.

2 METHODS

2.1 Experimental Setup

Three-point bending tests were performed on snow samples of naturally deposited snow. The specimens were cut out of the snowpack in the surroundings of Davos (Switzerland) with rectangular aluminium cases. The density of the snow samples ranged from 150 kg/m^3 to 350 kg/m^3. Four different sizes of cases were used (see Table 1). The largest snow case was close to the limit of what can be handled in the field and transported to the laboratory without destroying the natural snow structure The other sizes were chosen such that a size range of 1:4 was achieved. The thickness w was chosen to be the same for all specimen sizes in order to avoid a possible thickness effect ("2D similarity", according to Bazant and Planas [9]).

All experiments were performed in the SLF cold laboratories at Weissfluhjoch and Davos (Switzerland). The snow specimens were placed on two aluminium cylinders in a standard testing apparatus. Into the central cross section, a sharp cut was introduced from below with a metal saw blade. The central load was applied in displacement control at a constant rate of 200 mm/min through another aluminium cylinder. The velocity was chosen rather high in order to avoid viscous effects and cause a brittle fracture. The aluminium cylinders were used to prevent an unwanted caving in the snow sample while testing. We expect the diameter of the cylinders and the contact area to the snow sample to have only a minor influence on the test results.

The force required to break the specimens, $F_{app,}$ ranged between 1 and 60 N, depending on beam size and density. Because the applied forces were relatively low, the weight of the beam had to be taken into account. Thus, the nominal strength σ_N. , i.e. the maximum surface stress in the uncracked state, is determined by:

$$\sigma_N = \frac{3s}{2h^2}\left(\frac{F_{app}}{w} + \frac{1}{2}\rho g s h\right) \tag{1}$$

where s is the distance between the two supporting points, h is the sample height, w is the sample width and g is the acceleration due to gravity. Applying linear elastic fracture mechanics, the fracture toughness in tension K_{Ic} is obtained as:

$$K_{Ic} = \sigma_N \sqrt{\pi a} \cdot F\left(\frac{a}{h}\right) \tag{2}$$

where a is the notch length and F a function depending on the ratio of notch length to sample height (a/h) (Tada, et al. [10]).

Table 1: Snow specimen dimensions.

Height h (cm)	Length l (cm)	Width w (cm)
8	20	10
13	31	10
20	50	10
32	80	10

2.2 Scaling Laws

It is well known that fracture processes that are governed by nominal stresses or plastic collapse are scalable, whereas those governed by crack instability are not. Accordingly, there is no size effect in σ_N for small specimens up to a certain size. For large sizes a pure brittle behaviour is dominant and structures fail at a fixed stress intensity factor, which depends on the absolute crack length. This general behaviour is schematically shown in Figure 1 (Bazant and Planas [9]), using a general size parameter D. Based on experimental data of tensile fracture, Bazant and Planas [9] propose the following general scaling-law for the nominal strength

$$\sigma_N = \frac{B\sigma_T}{\sqrt{1+D/D_0}} \tag{3}$$

where σ_T is the tensile strength, B is a dimensionless constant, and D_0 is a constant with dimension length. However, eqn (3) can only serve to predict the size effect of geometrically similar specimens or crack systems. To transfer the results obtained from laboratory specimens to a slope stability analysis, additional relations based on physical models accounting for the material behaviour in the fracture process zone are required.

3 RESULTS AND DISCUSSION

3.1 Estimate of fracture process zone

Linear elastic fracture mechanics (LEFM) is valid only as long as nonlinear material deformation is confined to a small region ahead of the crack tip (Anderson [12]). Roughly, the size of the fracture process zone R_c can be estimated by (Bazant and Planas [9]):

$$R_c \cong \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_T}\right)^2 . \tag{4}$$

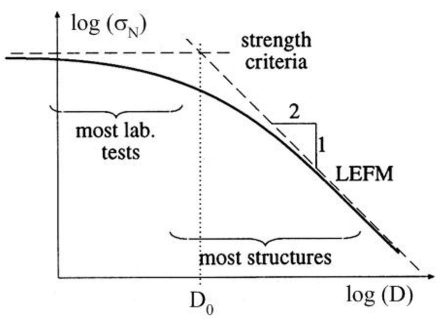

Figure 1: Fracture mechanical size effect on the strength of a material (Bazant and Planas [9])

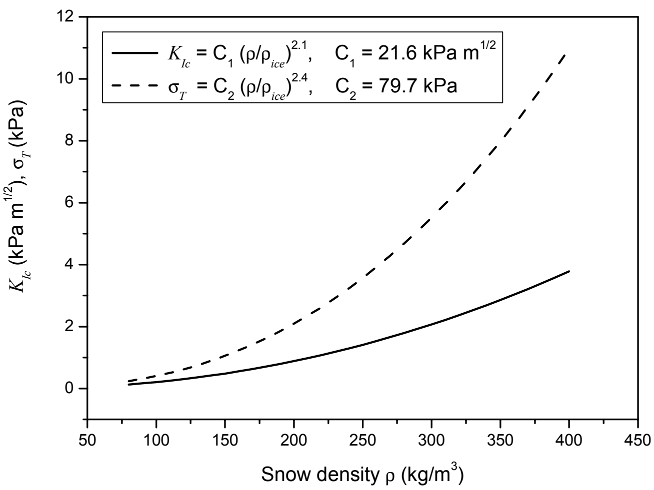

Figure 2: Fracture toughness K_{Ic} and tensile strength σ_T in relation to density. (Schweizer, et al. [3]; Jamieson and Johnston [13]).

The ratio of fracture toughness K_{Ic} to tensile strength σ_T was calculated from previous measurements of fracture toughness by Schweizer et al. [3] and of tensile strength by Jamieson and Johnston [13]. They fitted power law relations to their data with exponents of 2.1 for toughness and 2.4 for tensile strength (Figure 2). Accordingly, for a typical density of a dry snow slab avalanche of about 180 kg/m^3 eqn (3) delivers R_c to be about 6 cm. This means that the fracture process zone is in the same range as our specimen dimensions. Based on the data shown in Figure 2, the size of the fracture process zone decreases with increasing snow density. Non-linear fracture behaviour of a non-ductile material due to a relatively large fracture process zone is often called "quasi-brittle" (Bazant and Planas [9]). It is interesting to note that similar effects can be observed in quite different non-ductile materials like ceramics, if the specimen size is lower than a certain limit (Schindler [11]).

3.2 Preliminary results of fracture experiments

For the same specimen dimensions as used by Schweizer et al. [3] (20 cm x 50 cm x 10 cm) and the same linear elastic approach, , the values of K_{Ic} determined from the three-point bending tests turned out to be up to 50 % higher than the ones found by Schweizer et al. [3]. For, e.g., a density of 320 kg/m^3 the toughness K_{Ic} was 3.5 ± 0.7 kPa m^{½} whereas the power law fit proposed by Schweizer et. al [3] leads to 2.4 kPa m^{½}. Figure 3 shows our measurements in comparison with the power law fit for medium hard and fine grained snow proposed by Schweizer et al. [3]. The deviation seems to increase with increasing density. Because the same specimen size was used the difference between fit and three-point bending tests has to be attributed mainly to the different test methods (cantilever test vs. three-point bending test). As LEFM does not apply as discussed above, a shape effect on fracture toughness is not surprising.

First results of the fracture experiments with four different specimen sizes (Table 1) and constant ratio of cut length to specimen height (a/h) are shown in Figure 4. The slope of the linear regression in a bilogarithmic plot was -0.25 ± 0.07. As expected from the large size of the fracture process zone the slope of this curve deviates from -1/2 as associated with LEFM (Figure 1). Because of the small number of experiments ($N = 17$) this result has to be considered as preliminary. Neither LEFM nor the scaling law (3) seems to be capable to describe these test results. However, further experiments will be needed to corroborate this finding.

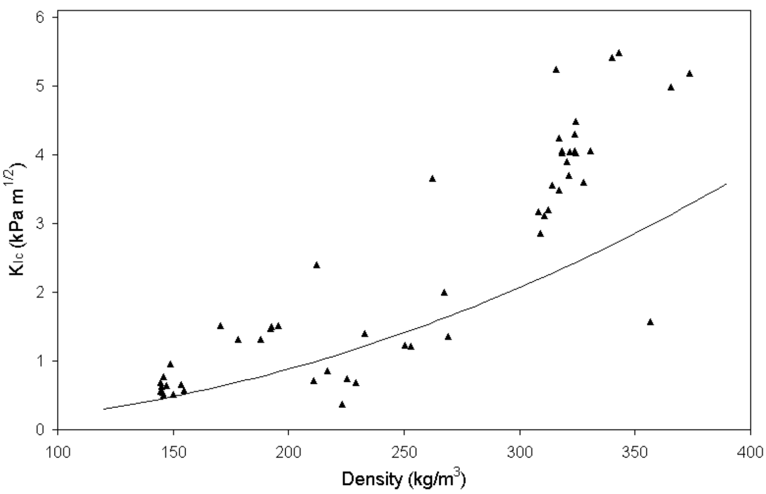

Figure 3: Three-point bending experiments: Snow fracture toughness in relation to density for a sample size of 20 cm x 50 cm x 10 cm. Toughness is calculated assuming linear elastic theory. The line is a power law fit proposed by Schweizer et al. [3] based on cantilever experiments for medium hard, fine grained snow.

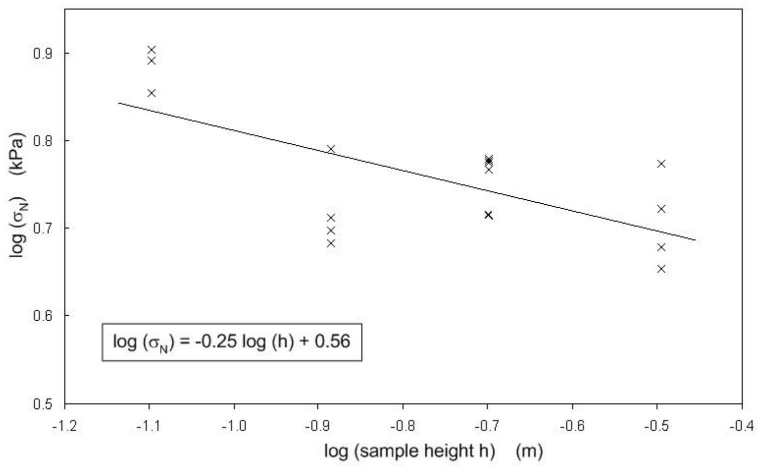

Figure 4: Bilogarithmic plot of nominal strength σ_N vs. sample height h. The linear regression was statistically significant (level of significance $p = 0.0024$) and had a slope of -0.25 ± 0.07.

4 CONCLUSIONS

Three-point bending tests on snow-specimens were performed to calculate the fracture toughness in tension K_{Ic}. The test method proved to be applicable for snow. Preliminary results suggest that the previously used cantilever beam experiments underestimate fracture toughness. The size of the fracture process zone in snow under tension was estimated to be of the order of centimetres, implying that snow has to be considered as a quasi-brittle material at the scale of our experiments. For a quasi-brittle material linear elastic fracture mechanics (LEFM) is applicable only with a size correction. Preliminary results showed that our laboratory experiments are too small to be in the linear elastic range. The manageable sample size is limited for practical reasons, because of the brittleness and softness of snow. Therefore, the effective fracture toughness obtained from these tests depends on size and shape of the specimen. Further experiments will be needed to quantify these effects in more detail and to find a suitable size correction. This will allow to make laboratory measurements on snow fracture toughness which will be applicable to snow slope stability and avalanche release modelling. However, the results indicate that even at the scale of a slab avalanche, fracture toughness is expected to be dependent on thickness of the snow layer, and the behaviour will be quasi-brittle as well, as pointed out by Bazant et al. [6].

REFERENCES

[1] Schweizer, J., Jamieson, J.B. and Schneebeli, M., Snow slab avalanche formation, *Reviews of Geophysics*, 41, 2003.

[2] Kirchner, H.O.K., Michot, G. and Suzuki, T., Fracture toughness of snow in tension, *Philosophical Magazine A*, 80, 1265-1272, 2000.

[3] Schweizer, J., Michot, G. and Kirchner, H.O.K., On the fracture toughness of snow, *Ann. Glaciol.*, 38, in press, 2004.

[4] Kirchner, H.O.K., Michot, G. and Schweizer, J., Fracture toughness of snow in shear and tension, *Scripta Materialia*, 46, 425-429, 2002.

[5] Kirchner, H.O.K., Michot, G. and Schweizer, J., Fracture toughness of snow in shear under friction, *Phys. Rev. E*, 66, 027103(3), 2002.

[6] Bazant, Z.P., Zi, G. and McClung, D., Size effect law and fracture mechanics of the triggering of dry snow slab avalanches, *J. Geophys. Res.*, 108, 2119, 2003.

[7] Dempsey, J.P., Defranco, S.J., Adamson, R.M. and Mulmule, S.V., Scale effects on the in-situ tensile strength and fracture of ice. Part I: Large grained freshwater ice at Spray Lakes Reservoir, Alberta, *International Journal of Fracture, 95*, 325-345, 1999.

[8] Dempsey, J.P., Adamson, R.M. and Mulmule, S.V., Scale effects on the in-situ tensile strength and fracture of ice. Part II: First-year sea ice at Resolute, N.W.T., *International Journal of Fracture, 95*, 347-366, 1999.

[9] Bazant, Z.P. and Planas, J., *Fracture and size effect in concrete and other quasibrittle materials*, CRC Press, Boca Raton, U.S.A., 616 pp, 1998.

[10] Tada, H., Paris, P.C. and Irwin, G.R., *The stress analysis of cracks handbook*, Paris Productions, MO, Saint Louis, 1985.

[11] Schindler, H.J. Size effects on fracture toughness of brittle materials, *International Journal of Fracture, 82*, R3-R10, 1996.

[12] Anderson, T.L., *Fracture mechanics: fundamentals and applications*, 2nd edn., CRC Press, Boca Raton, U.S.A., 688 pp, 1995.

[13] Jamieson, J.B. and Johnston, C.D., In-situ tensile tests of snowpack layers, *J. Glaciol.*, 36, 102-106, 1990.