PLASTIC INSTABILITY ANALYSIS OF THIN-WALLED TUBE UNDER TENSION-PRESSURE COMBINED LOADING

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ABSTRACT

In this paper, a plastic instability analysis of thin-walled tube which is simultaneously subjected to the combined loading of axial tension and internal pressure is conducted using finite element method. In the suchlike condition, a stress/strain ratio on the specimen is various depending on the loading histories. Therefore, both axial and pressure loadings have to be independently controlled so that the ratio of longitudinal and circumferential stresses could be kept constant. To enable this, a novel arc-length method with two different loading modes is proposed. The general arc-length method cannot be directly applied to the present problem in which loading modes may change due to the deformation and the solution depends on loading path. In the present study, both axial loading and internal pressure are taken into account and these two loadings are treated as independent quantities in numerical analysis by the presented arc-length method. A numerical example is conducted and a validity of the method is shown by comparing with the theoretical solutions. Additionally, it is demonstrated that the proposed method can evaluate the maximum axial loading and pressure independently and the histories of both loadings can be also computed with no difficulty even if they have a non proportional relation.

1 INTRODUCTION

Tube hydroforming (THF) is a novel forming technique which has gotten a lot of attention in plastic forming fields. In THF, thin metal tubes are subjected to both axial tension loading and internal pressure. Under such combined loading, a material point on tube specimen shows various stress state and a plastic instability occurs at a different stage of plastic deformation depending on stress/strain path. Plastic instability is one of the very important phenomena to trigger material failure. In material forming, precise prediction of plastic instability is very useful way to improve accuracy of process. Instability analyses of a cylindrical tube under internal pressure have been conducted by Mikkelsen and Tvergaard [1]. In the study, only the internal pressure is considered without any additional axial force and therefore the stress/stain path is unique. Some studies consider not only pressure but also axial loading (Asnafi [2], Asnafi and Skogsgardh [3] and Xing and Makinouchi [4]), however, the axial loading is taken into account by controlling displacement or the ratio between axial loading and pressure is kept constant. Therefore, the loading modes are always unique in these studies.

Waszczyszyn and Cichon [5] conducted elastic buckling analyses under multiple loading modes. In the analysis, it is supposed that each loading mode is constant during deformation and problem is independent from loading path. Therefore, it cannot be applied to the present problem in which loading modes may change due to the deformation and the solution depends on loading path.

In the present study, both axial loading and internal pressure are taken into account and these two loadings are controlled as independent quantities in numerical analysis. For that purpose, an arc-length method with two different loading modes is proposed. A numerical example is conducted and a validity of the method is shown by comparing with the theoretical solutions.

2 NUMERICAL PROCEDURES

The arc-length method is a very popular scheme to solve a problem in which loading amplitude is unknown variable like buckling. When an external force vector $\boldsymbol{F}$ is supposed to be proportional to a reference loading mode $\boldsymbol{f}_{\mathrm{ref}}$, an equilibrium equation is written using a multiplier λ as

$$\boldsymbol{Q}(\boldsymbol{U}) = \boldsymbol{F} = \lambda \boldsymbol{f}_{\mathrm{ref}} . \tag{1}$$

$\boldsymbol{Q}$ is an internal force vector which is function of displacement vector $\boldsymbol{U}$. $\boldsymbol{f}_{\mathrm{ref}}$ may change during the deformation like pressure loading (Noguchi and Ishihara [6]). Since λ is added to the equilibrium equation as an additional unknown variable, another constraint condition must be introduced to solve eqn (1) and many techniques are proposed. One of them is proposed by Ramm [7];

$$\boldsymbol{r}^{(1)\mathrm{T}} \Delta \boldsymbol{r}^{(i)} = 0 , \tag{2}$$

$$\boldsymbol{r}^{(i)} = \left[\boldsymbol{U}^{(i)}; \lambda^{(i)}\right], \tag{3}$$

$$\Delta \boldsymbol{r}^{(i)} = \left[\Delta \boldsymbol{U}^{(i)}; \Delta \lambda^{(i)}\right]. \tag{4}$$

Here superscripts show number of iteration and variables with Δ denote incremental quantities from the last convergence point. $\boldsymbol{r}^{(1)}$ can be determined by $\boldsymbol{r}^{(1)\mathrm{T}} \boldsymbol{r}^{(1)} = S_0^2$ (Riks [8] and [9]) and S_0 is a scalar corresponding to the arc-length. Solving eqns (1) and (2) simultaneously, displacements $\boldsymbol{U}$ and the multiplier λ can be obtained. If there are two different loading modes I and II, the right hand side of eqn (1) is replaced by

$$\boldsymbol{Q}(\boldsymbol{U}) = \boldsymbol{F} = \lambda_{\mathrm{I}} \boldsymbol{f}_{\mathrm{I}} + \lambda_{\mathrm{II}} \boldsymbol{f}_{\mathrm{II}} . \tag{5}$$

To solve eqn (5), one more constraint is needed because there are two unknown multipliers. In the present study, the external force consists of axial loading and internal pressure and amplitudes of them are unknown scale factors. Then the equilibrium equation is

$$\boldsymbol{Q}(\boldsymbol{U}) = \boldsymbol{F} = T \boldsymbol{f}_{\mathrm{ax}} + P \boldsymbol{f}_{\mathrm{pr}} , \tag{6}$$

$$\boldsymbol{f}_{\mathrm{ax}} = \boldsymbol{L}(\boldsymbol{l}) \quad on\ S_{\mathrm{ax}} , \tag{7}$$

$$\boldsymbol{f}_{\mathrm{pr}} = -\boldsymbol{N}(\boldsymbol{n}) \quad on\ S_{\mathrm{pr}} . \tag{8}$$

$\boldsymbol{f}_{\mathrm{ax}}$ and $\boldsymbol{f}_{\mathrm{pr}}$ are the loading modes corresponding to axial loading and internal pressure. $\boldsymbol{L}$ and $\boldsymbol{N}$ are the discrete vectors considering the longitudinal direction $\boldsymbol{l}$ and the normal vector to the inner surface $\boldsymbol{n}$. S_{ax} and S_{pr} are the surfaces of the end of tube and the inner wall. To determine the two unknown variables T and P, the following scheme is proposed. When a thin-walled tube subjected to axial loading and internal pressure is under uniform deformation, T and P are given by

$$T = 2\pi \sigma_{\phi} R H , \tag{9}$$

$$P = \sigma_{\theta} H / R , \tag{10}$$

with longitudinal stress σ_{ϕ} and circumferential stress σ_{θ}. R and H are current radius and thickness of the specimen. $\alpha = \sigma_{\theta} / \sigma_{\phi}$ is defined and the relation between T and P is

$$T / P = 2\pi R^2 / \alpha . \tag{11}$$

Therefore, if α is given, it can be utilized as the new constraint condition and T and P can be simultaneously computed by eqn (6) with eqns (2) and (11).

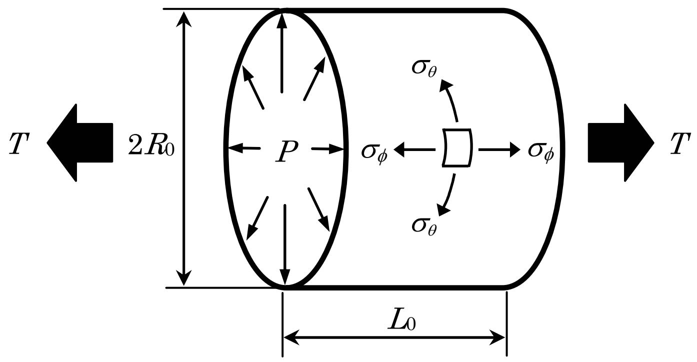

Figure 1: Schema of analysis model.

3 NUMERICAL EXAMPLE

3.1 Analysis model

The proposed method is implemented in a finite element analysis code. A schema of analysis model is shown in figure 1. The initial dimensions of specimen are $L_0/R_0 = 1.0$ and $H_0/R_0 = 0.1$ with the initial length L_0 , radius R_0 and thickness H_0. As for constitutive model, the general J_2-flow rule is adopted and a strain hardening law is defined as

$$g = \sigma_0\left[1 + \left(\varepsilon^{p}/\varepsilon_0\right)\right]^n , \tag{12}$$

$$\varepsilon^{p} \equiv \int_0^t \sqrt{2/3}\left(\boldsymbol{D}^{p} : \boldsymbol{D}^{p}\right)^{1/2} dt . \tag{13}$$

Here $\boldsymbol{D}^{p}$ is a plastic part of the deformation rate tensor. σ_0 , ε_0 and n are set to 400[MPa], 0.002 and 0.1 respectively. It is assumed that Young's modulus E = 200[GPa] and Poisson's ratio ν=0.3 as the elastic constants. 4-node isoprametric membrane element is applied and a quarter model is used because of the symmetric condition. The model is divided 1 and 20 elements along longitudinal and circumferential directions respectively. The stress ratio is kept constant from no loading state. In the present paper, such stress condition is called linear stress path.

3.2 Results

Figure 2 shows the equivalent plastic strain at the maximum loading points under each stress ratio. In the figure, the solid and chained lines denote theoretical solutions and the results obtained by the proposed method coincide with them. In the theoretical solutions, both of the axial loading and the internal pressure simultaneously reach to the maximum points when $\alpha = 1/2\left(\sigma_\phi : \sigma_\theta = 2:1\right)$ and the numerical result agree with the theoretical solution.

Figure 3 shows the loading histories with respect to the equivalent plastic strain under two stress paths. All histories are normalized by

$$\overline{T} = T/\left(2\pi R_0 H_0 \sigma_0\right), \tag{14}$$

$$\overline{P} = \left(P R_0\right)/\left(H_0 \sigma_0\right). \tag{15}$$

× denotes the maximum loading points. In case of $\alpha = 1/3\left(\sigma_\phi : \sigma_\theta = 3:1\right)$, the axial loading arrives the maximum point first and the pressure follows it. On the other hand, in case of $\alpha = 5/6\left(\sigma_\phi : \sigma_\theta = 6:5\right)$, the pressure reaches the maximum point in advance of that of axial loading follows. In both cases, after the maximum loading points, the loading histories

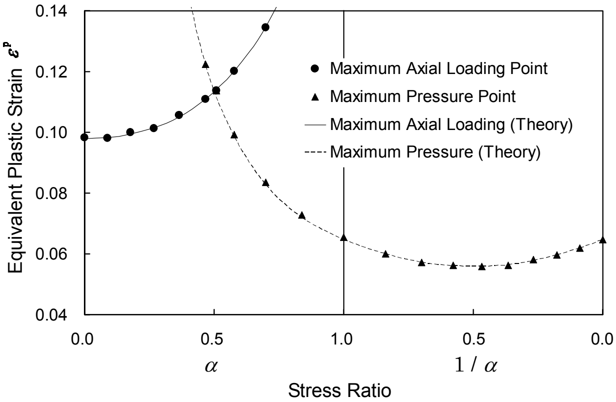

Figure 2: Equivalent plastic strain at maximum loading points.

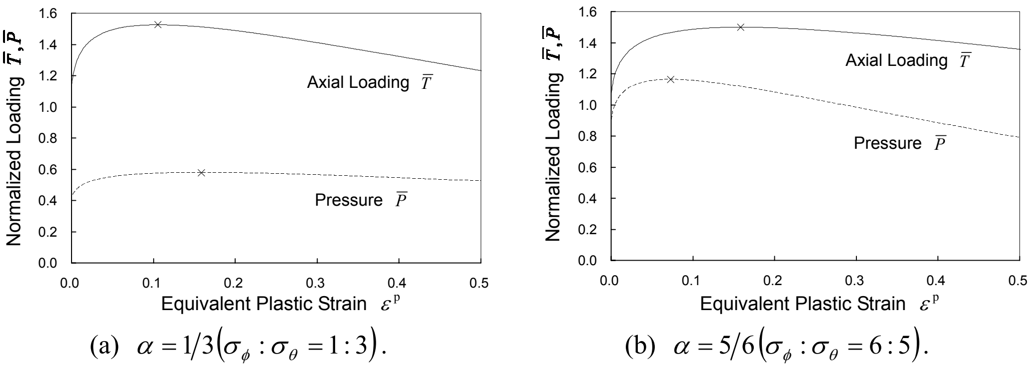

Figure 3: Loading histories with respect to equivalent plastic strain.

monotonically decrease. It is shown that the proposed method can evaluate the maximum loading points of both loading independently and trace the histories after it.

4 CONCLUSIONS

The arc-length method with two different loading modes is presented and the analysis of thin-walled tube subjected to the combination of axial tension loading and internal pressure is conducted. The numerical example demonstrates that the proposed method can evaluate the maximum axial loading and pressure of thin-walled tube under combined loading independently. Additionally, the obtained results coincide with the theoretical solutions. The histories of both loadings can be also computed with no difficulty even if have a non proportional relation. In the present method, it is not necessary that the deformation is uniform and it can be adopted in a strain localization analysis which defined as a concentration of plastic deformation caused by a collapse of uniformity of deformation. In future work, an evaluation of general failure criterion of tube specimen under combined loading should be conducted by the proposed method.

ACKNOWLEDGEMENT

This work is supported in part by Grant in Aid for the 21st century center of excellence for "System Design: Paradigm Shift from Intelligence to Life" from Ministry of Education, Culture, Sport and Technology in Japan.

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