Fracture Mechanics Analysis of Notch Fatigue of A Single Crystal Superalloy—CMSX-4

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EXTENDED ABSTRACT

Analytical and numerical studies of the fracture of single crystal superalloys have been performed. A closed-form solution for line cracks in single crystal Ni-base superalloys has been derived based on the continuously distributed dislocation theory (CDDT) for anisotropic materials. Then, finite element analysis is performed for semi-elliptical cracks embedded at the notch root of a single crystal Ni-base superalloy. A correlation has been found between the J-integral (or energy release rate) of the semi-elliptical crack and that of the line crack, under small-scale yielding conditions. Under such conditions, the J-integral can be expressed in terms of stress intensity factors but it includes an orientation dependent elastic constant, which has to be solved from the characteristic eigenvalue equations of the material. This correlation can be used to establish the fracture mechanics basis of fatigue crack growth analysis for single crystal Ni-base superalloys.

The above fracture mechanics solutions are applied to the analysis of the fatigue behavior of CMSX-4 notched specimens. The single crystal superalloy exhibits an order-of-magnitude difference in its fatigue life when tested under the same cyclic loading condition. The scatter has been attributed to the geometry and location of inter-dendrite pores, which were primarily crack initiation sites. Assuming that the initial crack starts from an interdendritic pore of elliptical shape, then the above fracture mechanics solutions can be used to describe the propagation of the crack. It shows that the crack growth profile tends to go circular from its original elliptical shape. This simulated profile corroborates with the experimental observation on the fracture surface, and the growth of internal cracks takes a major portion of the total fatigue life of the specimen. The calculated crack growth life is in good agreement with the fatigue-tested life. This fracture mechanics approach sheds light onto life prediction of single crystal components as opposed to the “strain-life approach”.

Introduction

Single crystal Ni-base superalloys have been widely used in advanced gas turbine engines as turbine blade and vanes. Studies of the fracture of these single crystal materials were mainly based on the linear elastic fracture mechanics (LEFM) and machine notched long crack fatigue crack growth testing (Doeker[1], Okazaki[2], Antolovich[3], Chan[4], Reed[5]). A few theoretical/numerical LEFM analyses have been conducted for cracks in single-crystal or generally anisotropic materials (Sih[6], Wu[7]), and the results indicate that the stress intensities in anisotropic materials are essentially identical to that in isotropic materials for the same crack size under equivalent loading conditions. Yet, the crack growth behavior and the fatigue life of a single crystal superalloy depends on the orientation. That is to say, in terms of the Paris relationship, $da/dN = C\Delta K^m$, the proportional constant C and the power index m are orientation dependent material constants. Then, the question is through what physical property of the material the orientation dependence is introduced? Apparently the answer does not lie in LEFM itself.

In this paper, the first section introduces an elastic-plastic fracture mechanics interpretation based on the continuously distributed dislocation theory (CDDT) for anisotropic materials. A closed-form solution for line cracks in generally anisotropic materials is presented. The next section describes a finite element modeling of a semi-elliptical crack at the notch root of a single crystal specimen. The numerical analysis is then related to the previous analytical formula with the equivalence of a surface crack to a through the thickness crack as its length divided by its shape factor Q. Finally, the above-developed concepts is applied to analyze the fracture behavior of notched specimens of CMSX-4 under fatigue conditions.

CDDT

Assuming that a line crack consists of dislocation pile-ups with the resultant stress satisfying the crack surface boundary condition, following the Bilby-Cottrell-Swinden approach (Bilby[8]) and using the Stroh formalism (Stroh[9]), the crack opening displacement and energy release rate of a crack in an anisotropic elastic-perfectly-plastic material under mode I loading condition has been derived (Wu[10]), as:

$$u_2 = \frac{4a}{\pi} t_2^F F_{22}^{-1} \ln \frac{c_2}{a} \qquad G = \frac{4a}{\pi} (t_2^F)^2 F_{22}^{-1} \ln \frac{c_2}{a}, \tag{1}$$

where F_{22} is an element of the material's elastic matrix solved according to Stroh formalism [9] and c_2 is the dislocation distribution length, as given by

$$\ln \frac{c_2}{a} = \ln\left(\cos \frac{\pi t_2^0}{2 t_2^F} \right)^{-1}. \tag{2}$$

In equation (2), t_2^0 (= σ) is the applied stress and t_2^F (= σ_s) is the yield strength of the material in the tensile direction.

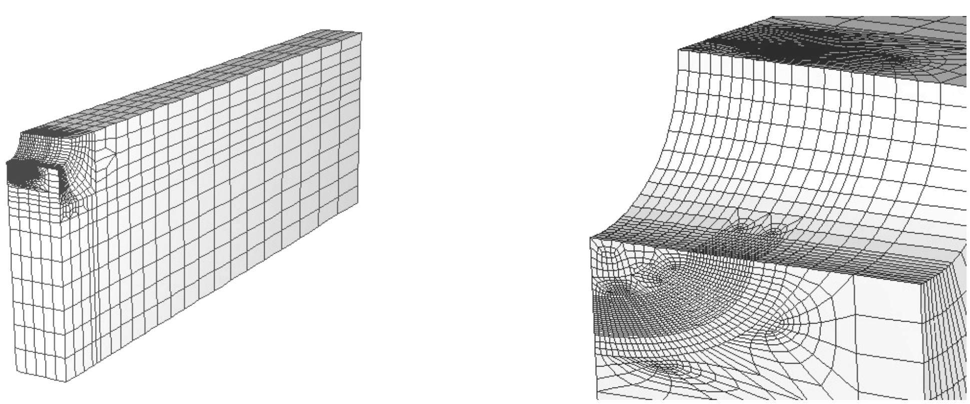
FEM

A semi-elliptical crack at the notch root of a single crystal specimen is modeled using finite element method, as shown in Figure 1. Also, assuming the elastic-perfectly plastic yielding condition for the material, the energy release rate (or the J-integral) is calculated. Its variation as function of the applied stress is shown in Figure 2.

On the other hand, by setting the equivalence of a semi-elliptical crack with a through the thickness crack, as

$$K = F_{sn} \sigma \sqrt{\pi \bar{a}} \;=\; K = F_{sn} \sigma \sqrt{\frac{\pi a}{Q}} \tag{3}$$

and substituting $\bar{a} = a/Q$ into Eq. (3), one can calculate the theoretical J for semi-elliptical crack at the notch root as a line crack under the stress multiplied by the strain concentration factor. A good agreement is found between the analytical and the numerical approach, as shown in Figure 2 (Zhao[11]), while the former obviously provides a great deal of convenience for analysis of cracks in single crystal materials.



(a) (b)

Figure 1: FEM model of ¼ notched specimen containing a semi-elliptical crack;
(a) ¼ notch cracked bar and (b) details of the notch root

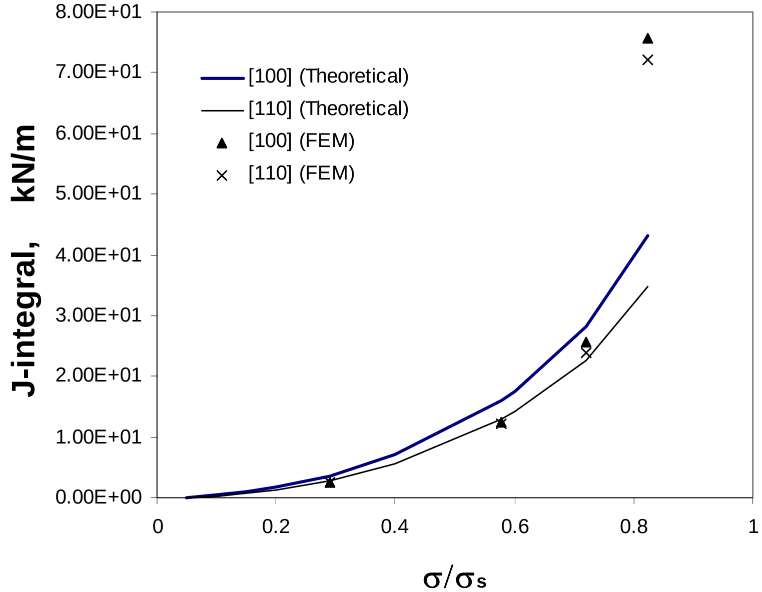


Figure 2: J-integral as function of applied stress.

Notch Fatigue of CMSX-4

In fatigue fracture of single crystal CMSX-4, the crack initiation sites were found to be primarily inter-dendrite pores at subsurface locations, and initially crack growth occurred as expansion of those internal material defects (Miller[12]). A scanning electron microscopy image of the fracture surface is shown in Figure 3. With regards to these observations, the following assumptions are made:

1. The inter-dendrite pores are assumed to be initial cracks of elliptical or semi-elliptical shape, depending on whether they are at subsurface locations or on the surface. The major/minor axes of these cracks are all lying in the [100] direction, by the nature of formation of inter-dendrite pores.
2. The growth of such internal (subsurface) cracks is controlled by crack growth in the [100] direction (along the major/minor axes) in a vacuum-like environment, such that the expansion of the crack remains elliptical.
3. The growth of internal cracks is assumed to be in a constant stress (the stress at the location of the pore in the ideal continuum body) field, because these cracks were found within the plastic yielding region close to the notch surface.
4. Once the crack breaks through to the specimen surface, it is assumed to take a half-penny shape with a radius equal to the diameter of the original internal crack, and the subsequent crack growth is considered as a long crack under the influence of a gradient stress field in the actual specimen.

The stress analysis by the finite element method (FEM) indicates that there exists a small-scale yield zone at the notch root, where the stress appears to be uniform. Since most fracture origins have been found by fractographic examination to be pores close to the surface, the condition of assumption 3 is reasonable. FEM analysis has also shown that the J-integral of a notch root crack can also be expressed in terms of the stress intensity factor, despite plasticity occurring at the notch root (Zhao[11]).

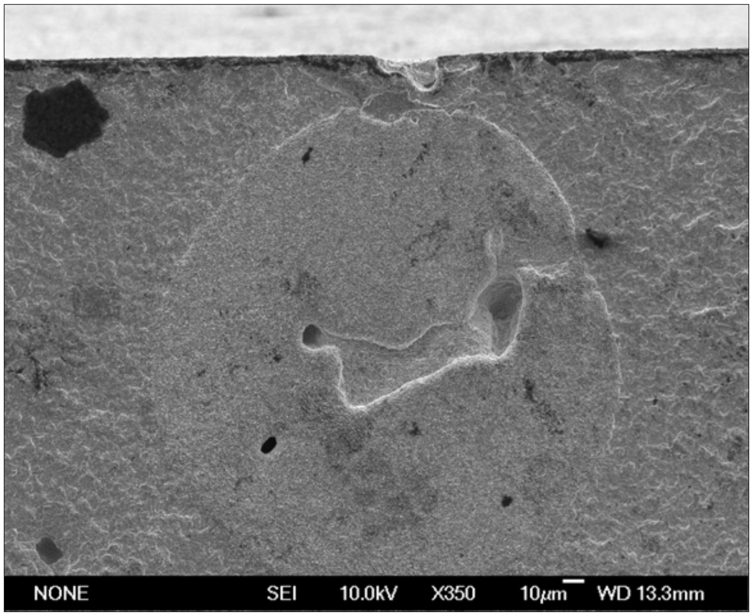


Figure 3: Sub-surface initiation in CMSX-4 at 650°C after 6,500 cycles.

Specimens of two crystallographic orientations, A: (001)[110] and B: (001)[100] (crack plane/direction) have been tested. Fatigue crack growth rates were found to behave as the Paris relationship, i.e., da/dN = $C\Delta K^n$ [4]. At 650°C, in vacuum $C_{[100]} = 1.85\times10^{-9}$ mm, while in the air $C_{[100]} = 2.46\times10^{-10}$ mm, $C_{[110]} = 2.25\times10^{-10}$ mm (the scale is normalized with a unit stress intensity factor). The difference between $C_{[100]}$ in the air and vacuum can be attributed to environmental effects such as oxidation, but the ratio between $C_{[100]}$ and $C_{[110]}$ is found to be approximately equal to the ratio of F_{22} for the respective orientation, indicating that despite oxidation may chemically change the crack-tip material, the crack growth rate is still partially controlled by the CTOD of the base superalloy.

Notch fatigue crack growth in CMSX-4 was simulated using an in-house computer code. The failure point was taken as the stress intensity factor reaches the critical value of 60 MPa√m. The numerical simulation shows that the crack shape would tend to become circular, as the internal crack expands, before it breaks through to the surface, which agrees with the postmortem fractographic observation. The total numbers of cycles to failure are given in the column of "Calculated life" in Table 1. In most cases of crack growth starting at an internal pore, the majority of the fatigue life is consumed in internal crack (or small crack) growth, as indicated by the number in the bracket. Once the crack breaks through to the surface, the remaining life is relatively short. The overall agreement of the calculation with the experiments is good, except in a few cases where there are differences by a factor of 2~3, which is most common in fatigue life scatter.

Table 1 Notch Fatigue Life of CMSX-4

Orientation /Test No.	P_{max} (kN)	Major axis (μm)	Minor axis (μm)	Depth (μm)	Test life	Cal. Life
X/3	6.2	100	10	80	62,000	22,800 (17,000)
B/4	6.2	90	10	130	6,500	11,100 (4,600)
A/6	6.2	100	25	250	25,500	49,000 (30,000)
A/8	6.2	40	15	0	21,661	32,000
A/9	6.2	50	30	200	5,270	6,500 (4,000)
B/11	6.2	50	5	350	13,717	14,900 (12,800)

*Note that the number in the bracket indicates the life of internal crack growth.

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