THEORETICAL FRAMEWORK FOR COUPLING OF NON-LOCAL DAMAGE AND VISCOPLASTICITY FOR DYNAMIC LOCALIZATION PROBLEMS

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ABSTRACT

Conventional continuum mechanics models of inelastic deformation processes are size scale independent. In contrast, there is considerable experimental evidence that inelastic flow in crystalline materials is size-dependent. At present there is no generally accepted framework for analyzing the size-dependent response of an inelastically deforming material. This is due to the fact that very limited quantitative numerical comparisons with experimental results were conducted, particularly, in localization problems. As soon as material failure dominates a deformation process, the material increasingly displays strain softening and the finite element computation is considerably affected by the mesh resolution. Several localization limiters that incorporate length scale measures in the constitutive relations have been successfully used in the literature to remove the inherent mesh sensitivity of the numerical failure predictions and to solve size scale dependency. This study develops a general consistent and systematic framework for the analysis of heterogeneous media that assesses a strong coupling between viscoplasticity and anisotropic visco-damage for dynamic problems. Since the material macroscopic thermo-mechanical response under dynamic loading is governed by different physical mechanisms on the meso- and macro-scale levels, the proposed model is introduced with manifold structure accounting for discontinuous fields of dislocation, crack, and void interactions. The gradient theory of rate-independent plasticity and rate-independent damage that incorporates macro-scale interstate variables and their higher-order gradients is generalized here for rate-dependent plasticity and rate-dependent damage to properly describe the change in the internal structure and in order to investigate the size effect of statistical inhomogeneity of the evolution-related rate- and temperature dependent materials. The idea of bridging length-scales is made more general and complete by introducing spatial higher-order gradients in the temporal evolution equations of the internal state variables that describe hardening in coupled viscoplasticity and visco-damage models, which are considered here dependent on their local counterparts. Computational issues concerned with the current gradient-dependent formulation of initial-boundary value problems are introduced in a finite element context. This framework is used to study among others the effect of material length scales on the localization of inelastic flow in shear bands.

1 INTRODUCTION

Generally, the nonlinear material behavior may be attributed to two distinct material mechanical processes: *plasticity* (i.e. dislocations along crystal slip planes) and *damage* (cracks, voids nucleation and coalescence, decohesions, and cleavage in regions of high stress concentration); see Figure 1. These two degradation phenomena are described best by the continuum theories of plasticity and damage mechanics. However, as the plasticity and damage defects localize over narrow regions of the continuum, the characteristic length-scale governing the variations of those defects and their average interactions over multiple length-scales falls far below the scale of the local state variables of classical plasticity and damage theories used to describe the response of the continuum. This leads to the loss of the statistical homogeneity in the representative volume element (RVE) and causes strong scale effects; in such a way that all the macroscopic response functions of interest (e.g. the Helmholtz free energy, Ψ; the dissipation potential, Π; the Cauchy stress tensor, $\boldsymbol{\sigma}$; the small strain tensor, $\boldsymbol{\varepsilon}$; the stiffness tensor; $\boldsymbol{E}$; etc.) are sensitive to the distribution, size, and orientation of the micro-, meso- and macro-structural defects within the RVE. The plasticity and damage evolution processes are, therefore, statistically inhomogeneous at scales smaller than the scale of interest. This suggests that the macroscopic inelastic deformations and failure are governed

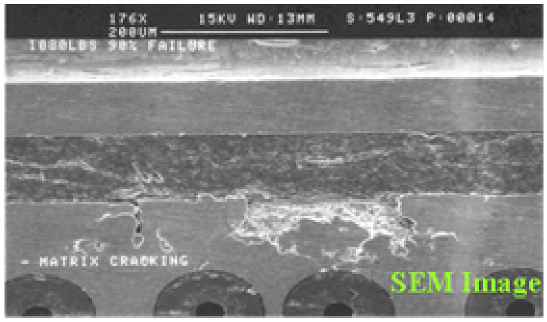

Figure 1: Heterogeneous damage evolution. After Voyiadjis et al. [1].

by mechanisms at different scale levels (*non-locality*) which gives rise to the *gradient-dependent* effects. Thus, the gradient effect is important when the characteristic dimension of the plastic and/or damage deformation zone is of the same order as the material intrinsic length-scale, which is in the order of microns for commonly used materials. For example dislocation interactions are observed on a meso-level with length-scale $0.1-10\mu m$ affecting strongly the material behavior on the macro-level with length-scale $\geq 100\mu m$.

Aifantis [2] was one of the first to study the gradient regularization in solid mechanics. Other researchers have contributed substantially to the gradient approach with emphasis on numerical aspects of the theory and its implementation in finite element codes: Lasry and Belytschko [3], Zbib and Aifantis [4]; de Borst and Mühlhaus [5]; Pamin [6]; Voyiadjis and Abu Al-Rub [7], etc. See Abu Al-Rub [8] for an extensive review of the non-local (integral type and gradient type) theories that appeared in the last three decades. Coupled Gradient thermodynamic plasticity-damage models were also introduced by Voyiadjis et al. [9, 10]. Moreover, we refer to interesting contributions on the physical origin of the length scale in gradient plasticity theories proposed by Voyiadjis and Abu Al-Rub [11] and Abu Al-Rub and Voyiadjis [12].

This paper addresses the problem of material length scales with the underlying objective to efficiently incorporate material length scales in the classical continuum plasticity and damage theories through the development of a gradient-dependent theory. This development will allow us to investigate a range of phenomena that cannot be well addressed by the conventional continuum plasticity and damage theories such as the size-dependence of material behavior and localization in softening media.

2 CONSTITUTIVE RELATIONS

The coupled constitutive model of viscoplasticity and ductile visco-damage used to predict material behavior under dynamic loading conditions is derived by Voyiadjis et al. [9, 10] based on a thermodynamic consistent framework. Thus, only the main equations will be given in the following. The model is based on the gradient plasticity and gradient damage theories. It includes the von Mises yield criterion, the non-associated flow rules, isotropic and anisotropic strain hardening, strain rate hardening, and softening due to adiabatic heating and anisotropic damage evolution. The von Mises criterion in the effective (undamaged) configuration is given by

$$f = \sqrt{3J_2} - [\bar{\sigma}_{yp} + \bar{\bar{R}}][1 + (\eta^{vp}\dot{\bar{\bar{p}}})^{1/m}][1 - (T/T_m)^n] \leq 0 \,, \tag{1}$$

where $J_2 = 1/2(\bar{\tau} - \bar{\bar{X}}):(\bar{\tau} - \bar{\bar{X}})$ is the second invariant of the resultant deviatoric stress tensor $(\bar{\tau} - \bar{\bar{X}})$, $\bar{\sigma}_{yp}$ is the initial yield strength (at zero absolute temperature, zero plastic strain, and static strain rate), T is the absolute temperature, T_m is the melting temperature, $\bar{\bar{R}}$ and $\bar{\bar{X}}$ are the non-local isotropic and anisotropic hardening stresses, respectively, $\dot{\bar{p}} = \sqrt{\frac{2}{3}\bar{\boldsymbol{d}}^{vi}:\bar{\boldsymbol{d}}^{vi}}$ is the rate of the effective accumulative viscoplastic strain, m and n are material constants, η^{vp} is the viscoplasticity relaxation time, and $\bar{\tau}$ is the average effective deviatoric stress tensor. This tensor is expressed in terms of the damage fourth-order tensor $\widehat{\boldsymbol{M}}$ and the corresponding damage state as follows:

$$\bar{\tau}_{ij} = \widehat{M}'_{ikjl}\sigma_{kl} \quad \text{with} \quad \widehat{M}'_{ikjl} = \widehat{M}_{ikjl} - \frac{1}{3}\widehat{M}_{rkrl}\delta_{ij}\,, \tag{2}$$

where $\widehat{\boldsymbol{M}}$ is the non-local fourth-order damage-effect tensor, which considers the distributions (spacing and orientation) and size of cracks and voids, and is given by

$$\widehat{M}_{ikjl} = 2\left[\left(\delta_{ik} - \widehat{\phi}_{ik}\right)\delta_{jl} + \delta_{ik}\left(\delta_{jl} - \widehat{\phi}_{jl}\right)\right]^{-1}. \tag{3}$$

Note that the subscripted indices indicates tensorial notation, and the superimposed hat and bar denote the spatial non-local and undamaged operators, respectively. $\widehat{\boldsymbol{\phi}}$ is the gradient-dependent damage (cracks and voids) density. Each non-local state variable is decomposed into local and gradient-dependent parts. The non-local evolution equations for the effective inelastic strain, $\widehat{p}$, and $\widehat{\boldsymbol{\phi}}$ are given, respectively, by

$$\dot{\widehat{p}} = \dot{p} + \ell_1^2\,\nabla^2\dot{p}\,, \quad \widehat{\phi}_{ij}^{\nabla} = \phi_{ij}^{\nabla} + \ell_2^2\,\nabla^2\phi_{ij}^{\nabla}\,, \tag{4}$$

where ℓ_1 and ℓ_2 are the plasticity and damage material length scales, respectively, and ∇^2 and $()^{\nabla}$ designate the Laplacian operator and corotational objective derivative, respectively. The non-local evolution equation for the isotropic hardening in the effective configuration, $\bar{\bar{R}} = \bar{R} + \bar{R}^g$, is given by

$$\dot{\bar{R}} = \frac{a_1}{\left(1-\widehat{r}\right)^2}\left(1 - k_1\bar{R}\right)\dot{\bar{p}}\,, \quad \dot{\bar{R}}^g = \frac{b_1\nabla^2\dot{\bar{p}}}{\left(1-r\right)^2}\left(1 - k_1\bar{R}^g\right). \tag{5}$$

The non-local evolution equation for the kinematic hardening in the effective configuration, $\bar{\bar{\boldsymbol{X}}} = \bar{\boldsymbol{X}} + \bar{\boldsymbol{X}}^g$, is expressed as follows:

$$\bar{X}_{ij}^{\nabla} = \widehat{M}_{ikjl}\widehat{M}_{mknl}\left(a_2\bar{d}_{mn}^{vp} - k_2a_2\dot{\bar{p}}\bar{X}_{mn}\right),\; \bar{X}_{ij}^{g\nabla} = \widehat{M}_{ikjl}\widehat{M}_{mknl}\left(b_2\nabla^2\bar{d}_{mn}^{vp} - k_2b_2\nabla^2\dot{\bar{p}}\bar{X}_{mn}^{g}\right). \tag{6}$$

The non-associative flow rules for the viscoinelastic rate of deformation $\boldsymbol{d}^{vi}$ and the damage variable $\boldsymbol{\phi}^{\nabla}$ are given by

$$d_{ij}^{vi} = \dot{\Lambda}^{vp}\frac{\partial f}{\partial\sigma_{ij}} + \dot{\Lambda}^{vd}\frac{\partial g}{\partial\sigma_{ij}}\,, \quad \phi_{ij}^{\nabla} = \dot{\Lambda}^{vp}\frac{\partial f}{\partial Y_{ij}} + \dot{\Lambda}^{vd}\frac{\partial g}{\partial Y_{ij}}\,, \tag{7}$$

where $\dot{\Lambda}^{vp}$ and $\dot{\Lambda}^{vd}$ are the viscoplastic and visco-damage multipliers, respectively. In eqn. (7) g is the visco-damage growth surface and is given by

$$g = \sqrt{\hat{Y}_{ij}\hat{Y}_{ij}} - [l_d + \hat{K}][1 + (\eta^{vd}\dot{\hat{r}})^{1/m}][1 - (T/T_m)^n] = 0\,, \tag{8}$$

where the non-local damage forces $\hat{\boldsymbol{Y}}$ and $\hat{K}$ are, respectively, characterizing the visco-damage evolution and the visco-damage isotropic hardening forces, l_d is the initial damage threshold (at zero absolute temperature, zero damage, and static strain rate), η^{vd} is the visco-damage relaxation time, and $\dot{\hat{r}} = \sqrt{\hat{\boldsymbol{\phi}}^{\nabla} : \hat{\boldsymbol{\phi}}^{\nabla}}$ is the non-local damage accumulation. The evolution equations for the non-local visco-damage isotropic hardening function $\hat{K} = K + K^g$ can be expressed as follows:

$$\dot{K} = a_3\left(1 - h_1 K\right)\dot{r}\,,\ \dot{K}^g = b_3\left(1 - h_1 K^g\right)\nabla^2\dot{r}\,. \tag{9}$$

The non-local damage conjugate force, $\hat{\boldsymbol{Y}}$, which is interpreted as the energy release rate, is defined as follows:

$$-\hat{Y}_{ij} = \frac{1}{2}\varepsilon^e_{rs}E_{mnkl}\varepsilon^e_{kl}\hat{M}_{manb}\frac{\partial \hat{M}^{-1}_{arbs}}{\partial \hat{\phi}_{ij}}\,. \tag{10}$$

The increase in temperature is calculated using the following heat equation:

$$\rho c_p \dot{T} = \Upsilon \sigma_{ij} d^{vi}_{ij}\ , \tag{11}$$

where ρ is the current density and Υ is the heat fraction.

NUMERICAL EXAMPLE

The following example demonstrates the performance of the proposed theoretical framework in solving the mesh size dependency. Let us consider a two-dimensional initial boundary value problem for a specimen of length 100 *mm* and width 20 *mm*. The bottom side of the specimen is fixed and the topside is movable. The loading is enforced by a velocity of $30 m/s$ that acts at the free end of the specimen for a period of $t_f = 700\ \mu s$. The constitutive parameters used in the computation are listed in Table 1.

Time increments of order 10^{-8} *s* satisfy the stability criteria. The numerical analysis was performed by implementing a material subroutine VUMAT in the environment of the explicit finite element program ABAQUS. The results presented below are focused on the distribution of effective plastic strain at the final states of localization. Figure 2 shows the effective plastic strain distribution at the end of localization ($t_f = 700\,\mu s$) for two meshes (left mesh: 15 x 50, right mesh: 25 x 70). It is well-known that in the classical plasticity and damage theories, the localization zones stays confined to the size of one element since no length scale is involved in the evolution of the shear band. This causes a finer element size to result in a smaller shear band thickness with higher peak strains. However, in Figure 1 one can easily observe the intense equivalent plastic strain distribution that shows the width and the location of the shear band development. Moreover, it can be seen that the thickness of the shear band and the magnitude of the effective plastic strain are the same for the two meshes. Therefore, the inclusion of the higher-order gradient terms with explicit length scales in the damage/plasticity can solve this mesh dependency.

Table 1: The material constitutive parameters.

$\rho = 7850\ kg/m^3$	$E = 185GPa$	$\nu = 0.3$	$\sigma_{yp} = 122.5\,MPa$
$c_p = 460\ J/kgK$	$\Upsilon = 0.9$	$\eta^{vp} = 0.01$ sec	$\eta^{vd} = 0.01$ sec
$n = 1$	$m = 1$	$T_r = 300K$	$T_m = 1500K$
$l_d = 0$	$\ell_1 = 5\mu m$	$a_1 = 400MPa$	$k_1 = 0.1MPa^{-1}$
$b_1 = 0.005N$	$\ell_2 = 7\,\mu m$	$b_2 = 0.25N$	$a_2 = 20\,GPa$
$k_2 = 15\,GPa^{-1}$	$a_3 = 0$	$b_3 = 0$	$h_1 = 0$

CONCLUSIONS

The proposed gradient approach introduces second-order gradients in the hardening variables (isotropic and kinematic) and in the damage variable. These higher-order gradients are considered physically and mathematically related to their local counterparts. Special care is used to properly account for the coupling between the state variable and its corresponding higher-order gradient.

Length-scale parameters are implicitly and explicitly introduced into the present dynamical formulism. An implicit length-scale measure is introduced through the use of the rate-dependent

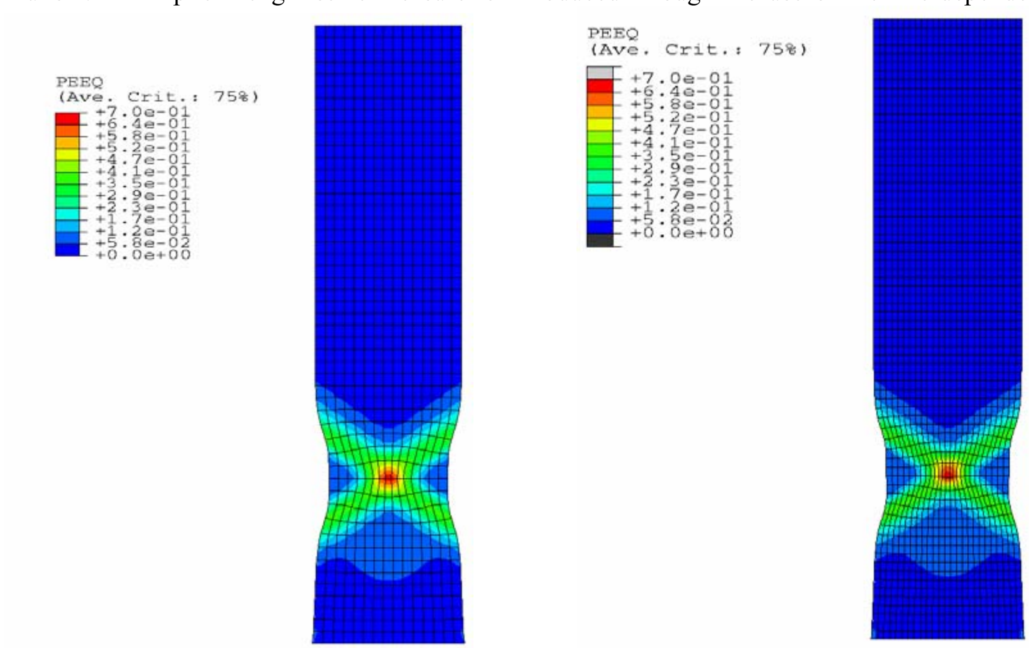

Figure 2: Mesh objective results (Left: 750 elements, Right: 1750 elements). Distribution of the equivalent plastic strain in a two-dimensional steel bar subjected to impact speed of $30m/s$.

theory, while explicit length-scale measures are introduced through the use of the gradient-dependent theory.

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