FRACTURE IN TWO DIMENSIONS (IN PAPER): ACOUSTIC EMISSION STUDIES AND THEORETICAL LESSONS

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ABSTRACT

Acoustic emission experiments in paper, a (quasi)-two dimensional disordered material, produce evidence of criticality, in the language of statistical physics. The energies of events follow a power-law, scale-free probability distribution. Two basic setups, usual mode I tensile tests and a nip-in-peel one have been studied. These are compared with each other, and with the the pertinent theories of slow fracture and crack line propagation.

1 INTRODUCTION

Acoustic emission (AE) is one of the statistical phenomena in fracture, that shows signs of universal phenomena, such as are encountered in many disguises in statistical physics (Lockner,Petri,Krysac,Guarino [1, 2]). The essential idea is that transient elastic waves are detected, once they get generated through the release of elastic energy due to microcracking and (main) crack propagation. The time-series at a single AE detector can be described by "crackling noise": silent intervals are separated by AE events of varying length and amplitude/energy (Bouchaud [3]). Numerous studies have elucidated the statistical laws that describe AE. In general, these relate to usual mode I or mode III -type loading conditions, and the common feature is that they exhibit *scalefree* features. The probability distribution function (pdf) of event energies is most often of power-law type, $P(E) \sim E^{\beta}$ with $\beta = 1 \dots 2$, and e.g. the event intervals are most often, similarly, found to obey such fat-tailed pdf's.

Other "fractal" characteristics relate to the properties of final crack surfaces. It has been discovered that both for 2d and 3d geometries fracture interfaces can be described by self-affine fractals (Bouchaud [4]). Thus e.g. the mean-square fluctuations w increase with the scale of observation as $w \sim l^{\chi}$, with the roughness exponent χ depending on the range of observation and dimensionality, among others. The reasons for scale-invariance are still being debated (Bouchaud, Bouchaud et al., Räisänen, Hansen and others [4, 5, 6, 7]). In this context, a very clean scenario is presented by the propagation of an interfacial crack (line) between two three-dimensional plates, of e.g. plexiglass. Now it is clear that if the crack front exhibits non-trivial fluctuations these could perhaps be accounted for by a dynamical model for such a line, in the presence of the right kind of *disorder*, and with the right kind of interactions (Schmittbuhl, Le Doussal, Rosso; Schmittbuhl, Alava [8, 9]). However, where as renormalization group calculations and numerical models indicate $\zeta \sim 1/3 \dots 0.38$, the experimental evidence points towards a much more complicated scenario (Schmittbuhl, Delaplace [10]). The roughness exponent is higher (upto 0.6), and there is clear evidence of the presence of localized activity, weak oscillations, or avalanches (Måløy [11]). In general, theoretical models for statistical fracture as found in the literature do not include more complicated effects than locally varying material properties (strength, elastic modulus) and simplified load-sharing. They do imply the presence of scale-free dynamics, and in the most simple case of democratic (global) load-sharing models (fiber bundle ones) (Kloster [12]), these properties can be

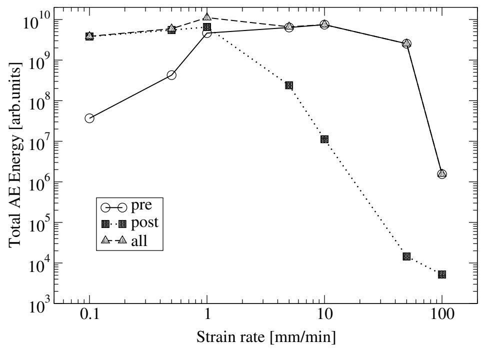

Figure 1: The fractions of AE energy, prior and after to the σ_c of the stress-strain curve, as a function of the strain rate in mode I experiments.

defined and measured exactly. In more elaborate systems, like random fuse networks (Herrmann [13]) which are a scalar analogy of quasi-brittle fracture, these features still persist (e.g. in terms of a broad $P(E)$ distribution, and the notion of avalanches) (Zapperi [14]).

Here we consider paper as a test case of theories of fracture in the presence of structural randomness. This is of interest since paper is (almost) two-dimensional, among others (Kertész [15]). In the following, we concentrate on two kinds of experiments (Salminen, 2002; Salminen et al. [16, 17]). The more recent is based on a "paper peeling", in which a crack line is forced to propagate along the sheet plane, thus separating the sheet into two halves. In addition, we consider the usual mode I fracture, and highlight the scaling by using the strain rate as an extra control parameter.

2 EXPERIMENTAL SETUP

Normal newsprint paper samples (size 100 mm by 100 mm) and laboratory sheets (size 70 mm by 15 mm) were tested in two geometries, in mode I (tensile) and peel-in-nip. The later produces very large fracture surfaces. Due to the lack of constraints the samples could have out-of-plane deformations in tensile tests, too, and none of the three fracture modes (I, II, III) is excluded on the microscopic level. The deformation rates $\dot{\epsilon}$ varied between 0.1 %/min and 100 %/min. During the experiment we acquire bi-polar acoustic amplitudes simultaneously by piezoelectric sensors, as a function of time. In most cases the AE apparatus consisted of two transducers, +27 dB amplifiers and continuous 12-bit data-acquisition. The time-resolution of the measurements was 2.5 μs and the data-acquisition free of deadtime. We made 20 identical repetitions for statistics. The strain rates are such that the sound velocity is much faster than the timescales implied by $\dot{\epsilon}$. The acoustic time-series are reformed offline by thresholding, detection of continuous and coherent oscillatory events, and the calculation of event energy E, the sum of squared amplitudes within the event. Events are separated by silent (i.e. amplitude below threshold level) waiting intervals τ. In general the energy of the event is expected to be proportional both to the damaged area and to the stress in that area.

3 TENSILE FAILURE

Here, one can compare the results with mean-field -like avalanche models (fiber bundles), that would imply using scaling $P(E) \sim E^{-(5/2+1)/2} = E^{-7/4}$, and simple simulations (Salminen 2002, Minozzi [16, 18]). One particularly interesting twist is demonstrated in Fig. 1, in which the accumulation of AE energy is divided according to whether it has originated, in a strain-controlled experiment,

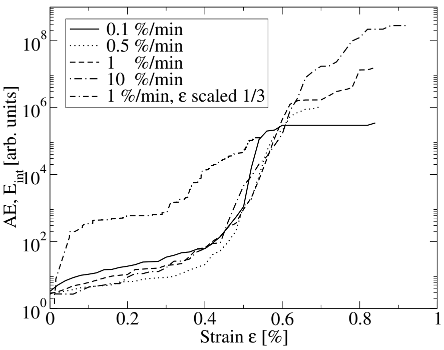

Figure 2: Integrated AE energy vs. strain, for four data sets with the same paper, but with different strain rates, and a fifth one as a comparison). Notice the exponential increase of the AE energy accumulation. The rounding-off of the curves signals the regime in which individual samples fail.

before or after the maximum stress σ_c. Excluding the last data point (for 100 mm/min) the typical total AE energy does not vary much, but the fractions do very much so. Since, the corresponding energy scalings imply in general $\beta \sim 1.2 \pm 0.1$, we must conclude that the microscopic crack growth dynamics is independent of the "macroscopic failure point", or whether catastrophic failure happens right at σ_c or beyond that. It is to be emphasized that in this respect it should not be of importance whether a test is made in stress- or strain-controlled circumstances. The acoustic emission energy accumulates close to the failure point in an exponential fashion, as depicted in Fig. 2.t There is evidence of a slow increase of a background AE level, which may be sensitive to particular experimental choices (thresholding etc.). and prior to the maximum stress the accumulation is fast. Attempts to match the AE integral with a power-law-like divergence ($\int E_{AE} dt \sim (\sigma_c - \sigma(t))^{-a}$) are unsuccessful, and note that the data exhibits several decades of exponential growth. We may therefore conclude, that there is no evidence of a "finite-time singularity" (Johansen, Shcherbakov,Guarino [19, 2]), in the fracture of paper.

4 CRACK LINE PROPAGATION IN A RANDOM ENVIRONMENT

In this case, the fracture dynamics presents steady-state conditions, unlike in a mode I experiment (say), and thus there are intriguing possiblities for the analysis of e.g. stationary AE timeseries. The first observation is that the dynamics is again scale-invariant (Salminen et al. [17]), and in both the AE energy and event-interval pdf's we see clear observations of fat-tailed, broad characteristics (Fig. 3). There are three main observations as mode I and in-plane fracture are compared: i) the β-exponent is much larger for the latter one. This may be attributed perhaps to the tendency, noted by various theoretical approaches, for the crack dynamics to be very much localized around the crack line on the expense of bulk damage ahead of it (Zapperi et al., Åström [20]). ii) A similar difference can be noticed in the waiting time statistics, as well, and iii) most importantly the in-plane peel test shows clear evidence for an *intrinsic scale*. Other tests at varying strain rates imply that the associated t_i derives from a fundamental length, closely related to the size of individual fiber-to-fiber bonds (30-50 μm). These, or the fibers themselves, are the fundamental building blocks of the structure (Salminen [17]). Finally, given the steady-state conditions the AE time series can be used to infer different kinds of correlations in the energy release. Theoretically, this is partly uncharted territory; in statistical physics avalanche models the avalanches (AE events in our language) are often rather uncorrelated, but can also be described by a broad pdf (eg. for the size or duration). In Fig. 4 we consider the autocorrelation function of the AE energy signal, after thresholding and

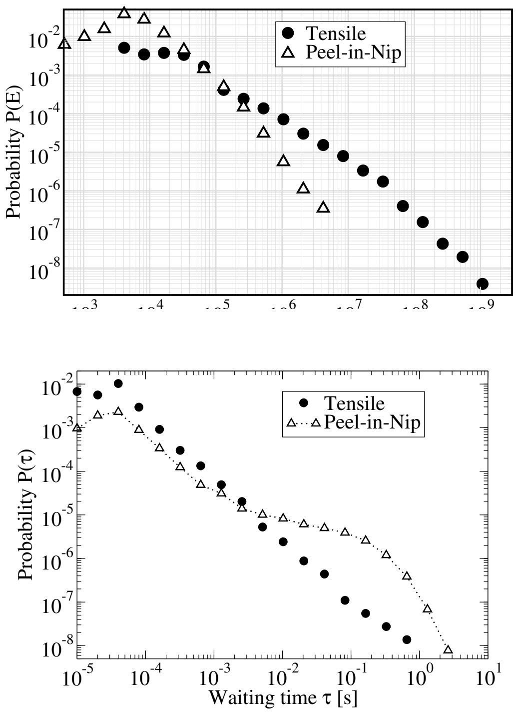

Figure 3: Top panel: the energy distribution $P(E)$ for two experimental setups. Both exhibit *power-law* statistics, but with different exponents, and that the mode I has a much smaller exponent (close to 1.2). Lower panel: The distribution of time intervals τ for the mode I and in-plane experiments. Both obey an Omori's law -like scaling, and the mode I one scales close to $P(\tau) \sim \tau^{-1}$. For the latter one there is a distinct time-scale (see text).

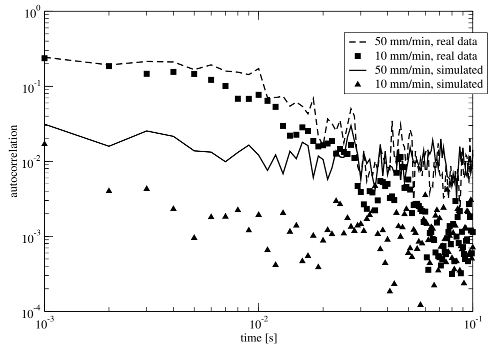

Figure 4: The autocorrelation function of released energy from the in-plane experiment, for two strain rates (10 and 50 mm/min). The data is compared to two artificial timeseries with the same interval and energy statistics.

dividing into discrete 1 ms timebins. The data is compared to two test sets, obtained by randomly rearranging the sequence of events and empty intervals. While the noise levels of the two strain rates are different (the average AE energy per bin increases with the rate), some conclusions can be drawn. The AE release is correlated upto a timescale that is only weakly dependent on the strain rate, upto about 0.02 . . . 0.08 s.

5 CONCLUSIONS

In this work, we have searched for universality in the fracture of paper, as both a test of current theories of statistical fracture and to present some landmarks for further theoretical work. In both the cases studied clear evidence is found of broad energy distributions. The two main conclusions have been: the mode I one is far smaller than that resulting from any model as fuse networks or fiber bundle models; and, in the peeling test a β-exponent is recovered, such that there is again no known theoretical framework. Meanwhile, the temporal statistics implies scale-invariance if measured for instance in terms of the event interval statistics. Both mode I and in-plane fracture, using the same material, imply correlations in the fracture process. In the development of damage prior to maximum stress in mode I we observe exponential growth of released energy. This is similar to random damage, or softening due to reduced elastic modulus, which leads to a localized fracture zone in quasi-brittle materials (ice, concrete etc.) (Van, Delaplace [21]). Unfortunately we do not have enough statistics to use "b"-analysis, the possible variation of the β-exponent along the stress-strain curve. This would also be a stringent test in the statistical physics sense, since many models imply that while the β-exponent would not change, the cut-off of the pdf would, thus resulting in a net β-exponent from the combination of these two. On the other hand, the data is clearly in contrast to attempts to describe the approach of σ_c in terms of the concept of a “critical point”. In the stationary statistics correlations can be seen, but it is unclear whether these result from structural correlations - the line getting “pinned” by stronger interfacial regions - or its intrinsic dynamical response. To summarize, paper fracture allows to test many simple scenarios of statistical fracture, and still more work could be done e.g. to consider the detailed properties or shape of the AE events in the various scenarios. A further prospect is to connect AE and damage dynamics (localization) to the roughness of the final crack line.

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