Extended Abstract

The Mechanics of Sawing Wood

by

Tony Atkins
Department of Engineering
School of Construction Management and Engineering
University of Reading
READING RG6 6AY
England
a.g.atkins@rdg.ac.uk

Extended Abstract

There are many empirical equations for the forces to cut wood under different conditions of processing using different types of cutting tool geometry. From such expressions, other empirical formulae for cutting power, cutting energy and specific cutting energy (energy to remove a unit volume of kerf) have been determined. For simple orthogonal two-dimensional cutting it is found that the cutting force F_H parallel to the direction of blade motion is given either by

$$F_H = K\, t^m\, w \tag{1}$$

for small chip thickness t, or by

$$F_H = (A + Bt)\, w \tag{2}$$

for thick chips. In both equations, w is the width of the chip and K, A, B and m are constants depending upon tool geometry and friction, and the mechanical properties of the wood. For example, for sugar pine having 8% moisture content cut parallel to the grain by a tool having 30° rake angle and 15° clearance angle, Franz (1958) gives

$$F_H/w = 180\, t^{0.4} \qquad \text{(lbs/in for t in inches)} \tag{3}$$

and

$$F_H/w = 22 + 730\, t \qquad \text{(lbs/in for t in inches)} \tag{4}$$

Where theory has been applied to explain the form of such empirical relationships, it has usually been of the Ernst-Merchant (eg 1944) type, originally proposed for orthogonal continuous chip formation in

metalcutting, in which deformation of the chip to flow up the rake face of the blade is assumed to be concentrated in a single (primary) shear plane orientated at an angle ϕ to the cut surface. Friction between the underside of the chip and the rake face of the tool is included and, from equilibrium of forces, expressions are obtained for F_H and F_V (the force perpendicular to the cut surface). The expressions for F_H and F_V are given in terms of the (unknown) angle ϕ. The 'best' value for ϕ is obtained by minimising the cutting work, which leads to

$$\phi = (\pi/4) - (1/2)(\beta - \alpha) \tag{5}$$

where β is the friction angle and α is the orthogonal tool rake angle. Unfortunately, this expression is independent of material properties, whereas it is known from experiments on metals, plastics, wood etc that ϕ is material dependent. Furthermore, the Ernst-Merchant line of attack predicts that plots of force vs uncut chip thickness (depth of cut) should pass through the origin whereas they most often have a positive force intercept at zero depth of cut (cf Equation (4)). Other considerations not covered by the simple analysis are the formation of discontinuous chips and splitting (the different types of chip in wood identified by Franz, equivalents of which occur with other materials, cf Rosenhain and Sturney, 1925). Wood's compressibility and severe anisotropy further complicate matters as discussed by Koch (1964, pp 79 et seq) for which modifications of the basic Ernst-Merchant analysis have been proposed.

The shortcomings of the Ernst-Merchant approach for cutting forces of all sorts of materials have been examined by Atkins (2003) in the light of both basic physical principles and the methodology required in finite element (FEM) simulations of cutting. In FEM modelling it is found that a 'separation criterion' has to be employed at the crack tip in order for the tool to cut a chip and move along the surface. While one view of such a requirement is that it is merely a computational convenience to avoid the singularity at the tip of the tool, it transpires that the *specific work* associated with many successful separation criteria employed for ductile metals are at the kJ/m^2 levels typical of fracture mechanics toughnesses, and that the form of the criteria are what would be expected from the micromechanisms of fracture by which the chip is separated from the surface. It was shown that inclusion of a surface work term, as well as the usual plasticity and friction components in even the simplest Ernst-Merchant model, resolved many of the previous shortcomings. For example, for given tool geometry and friction, ϕ becomes material dependent through the non-dimensional parameter $Z = R/\tau_y t$ where R is the fracture toughness, τ_y the shear yield stress and t the depth of cut. It was shown that

$$F_H/w = [R/Q] + [\tau_y \gamma /Q]t \qquad (6)$$

and

$$F_V = F_H \tan(\beta - \alpha) \qquad (7)$$

where $\gamma = \cot \phi + \tan (\phi - \alpha) = \cos \alpha/\cos (\phi - \alpha) \sin \phi$ is the shear stress on the primary shear plane and β is the friction angle on the rake face of the tool. $Q = [1 – \{\sin\beta\sin\phi / \cos(\beta - \alpha) \cos(\alpha - \phi)\}]$ is the friction correction factor. Solution of Equation (6) looks for the ϕ which minimises F_H.

For given tool geometry and friction, it is found that at large depths of cut (strictly below a critical Z of about $10^{-1} \sim 1$), the optimum ϕ is independent of t, from which it follows that the shear strain γ along the primary shear plane is constant and hence a linear relation between F_H/w and t is predicted which, however, does *not* pass through the origin and has a finite intercept (R/Q) which is a measure of the fracture toughness. This agrees with the empirical Equation (2). At small depths of cut which increase Z above the critical value, ϕ decreases as t decreases, and the shear strain γ increases; this produces a downwards curvature in plots of F_H vs t given by Equation (6) exactly as in the empirical Equation (1). However, unlike Equation (1), the curve given by Equation (6) again does *not* pass through the origin and has a finite intercept which is a different measure of the (same) R of the material. The two separate empirical relations for thin and thick chips given by Equations (1) and (2) are thus seen to be part of a single unified theory given by Equation (6).

Good agreement between the new theory and experiment has been found for orthogonal cutting of metals and plastics (Atkins, 2003a and 2003b) and wood (Atkins, 2004). In this paper we apply the theory to the particular results of McMillin and Lubkin (1959) relating to the climb-sawing of hard knot-free maple in different directions using an overcutting radial arm saw. The power consumed P is given by the average total cutting force M multiplied by the cutting velocity c, where the average total cutting force is the force F_H per tooth given by Equation (6) multiplied by the number of teeth engaged at any one time in cutting. The number of teeth cutting is b/p where b is the arc of contact between saw rim and material, and p is the pitch of the teeth. The mean thickness of cut t is given by [pfdk/bcg] for the spring-set teeth employed by McMillin and Lubkin, where f is feed rate, d is thickness material being cut, k is kerf width, and g is the thickness of the circular saw blade. In McMillin's and Lubkin's experiments all of p, c, k and g were held constant, so Equation (6) gives

$$P = C_1 b + C_2 f d \approx d (C_1 + C_2 f) \qquad (8)$$

since b ≈ d in practice. Reference to Equation (6) shows that

$$C_1 = [R k c /Q p] \text{ and } C_2 = [\tau_y \gamma k^2 /Q g] \qquad (9)$$

Equation (8) suggests a linear relation between P and f having positive intercept $C_1 d$ and slope C_2 d, which is exactly what was discovered experimentally, only now physical meaning can be put on the empirical constants. Making appropriate substitutions it is found that τ_y is some 25 MPa and R is about 100 J/m^2.

Experiments were performed at various angles θ to the grain orientation. The empirical expression obtained by McMillin and Lubkin for cutting power was

$$P = d (C_1 + C_2 f) (1 + E \sin^2\theta) \qquad (10)$$

where E was another empirical constant. The form of this relation is perhaps surprising, as it suggests that the same R/τ_y ratio applies for all orientations. Further discussion on this and other matters will be contained in the full paper.

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