NUMERICAL ANALYSIS OF INTERFACE FRACTURE IN TWO LAYER COMPOSITES UNDER MIXED MODE LOADING

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ABSTRACT

The numerical analysis of the interface crack growth in two layer composites under mixed mode loading is undertaken in this work. Two different R-curves are determined using two different total energy release rates based on the geometrically linear and non-linear thin plate theories. The blister test was utilised to provide critical pressure-crack length information for two composites. Constant R-curve behaviour for stiff/stiff composite system, and increasing R-curve tendency is observed for compliant/stiff composite. Results of the total energy release rate obtained from analytical expressions are compared with the finite element method results. It is also revealed that mode 2 prevails against the mode 1 contribution of the total energy release rate during the entire crack propagation range, as shown by the phase angle value.

1 INTRODUCTION

Composite laminates are used in a number of engineering applications ranging from microelectronics to structural engineering. Application of layered materials is strictly connected with the reliability of their interfaces' resistance to fracture (debonding, delamination) initiation and propagation. R-curves (Broek, 1986) provide a general information about the composite interface adhesion quality during crack propagation based on the knowledge of critical fracture parameters, e.g. the critical total energy release rate. Furthermore, the mixed mode fracture is inherently related to interface cracks between dissimilar materials, and it can be characterised by the phase angle value (Hutchinson and Suo, 1992). The information about the phase angle provides some basis for the formulation of an interface fracture criteria. The blister experiment and related closed form solutions of elasticity theory and fracture mechanics (Bennet et al., 1974; Hutchinson and Suo, 1992; Jensen, 1991; Williams, 1997) provide a very useful tool for characterisation and analysis of the interface crack growth in composite laminates.

The finite element method (FEM) (Zienkiewicz and Taylor, 2000) is often used to analyse the interface failure in cases where no analytical closed form solutions exist or to verify results obtained from existing analytical solutions (Figiel et al. (2004)), or determine critical fracture parameters (Beckert and Lauke, 1997). In other cases, the FEM based computational strategies can be used to simulate the interface crack growth (Roe and Siegmund, 2003), and thereby provide some useful results *a priori* experimental testing.

The main purpose of this work is the numerical investigation of the interface crack growth in two layer composites subjected to mixed mode loading. Two cases are studied: stiff/stiff and compliant/stiff composite systems. Two different analytical expressions are utilised to calculate the total energy release rate (TERR). The critical values of the TERR were determined with experimental results for these two composites, using the blister test. . These critical TERR results are used to determine the R-curve for crack propagation. The TERR results from analytical expressions are compared with TERR values obtained from the FEM results using ANSYS (ANSYS, 2002).

2 THEORETICAL BACKGROUND

The blister test specimen (Fig. 1) is modelled herein. A thin film (component 1) is attached to a substrate (component 2) with a hole. The pre-crack *a* at the interface of the two component laminate is considered. This interface crack is located between two elastic and isotropic materials, defined by the elasticity tensors $C_n=C_n(E_n,\nu_n)$ and thickness h_n; n=1,2. The lower component is stiff such that $E_1<E_2$ and supported in X_1 and X_2 directions. Thus, the stiffer material properties of the lower component are excluded from the solution of the boundary value problem (BVP). A uniform pressure *p* is always applied to the upper component over the delaminated region. The global rectangular co-ordinate system is $\mathbf{X}=\{X_1, X_2\}$, while the local rectangular co-ordinate system $\mathbf{x}=\{x_1, x_2\}$ is always attached to the interface crack tip. Two cases are distinguished (1) the upper component is stiff and (2) the upper component is compliant $h_1\to 0$. In the first case, it is supposed that there can occur either small deflections or strains during loading and crack propagation. In the second case, large deflections and rotations are allowed, while small strains might be expected. Therefore two concepts of plate theory are applied here to determine the total energy release rate (TERR): the linear and non-linear plate theories (Woźniak, 2002) along with the concept of linear elastic fracture mechanics for interface cracks (Suo and Hutchinson, 1990).

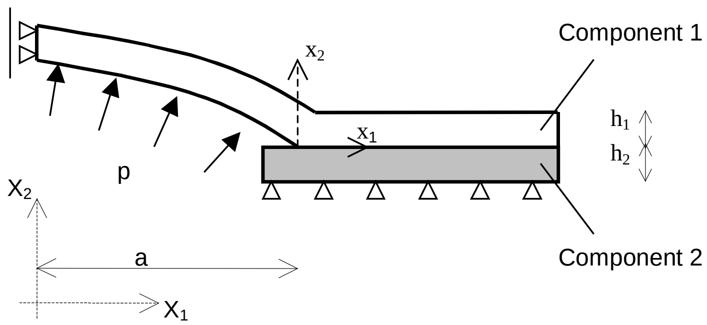

Figure 1: Two layer composite

The TERR for the case 1 can be determined based on the Clapeyron's theorem, Griffith's energy balance and the plate theory as follows (Bennet et al., 1974):

$$G_T = \frac{3\left(1-\nu_1^2\right)p^2 a^4}{32 E_1 h_1^3} \tag{1}$$

The modelling of the large deflection and/or rotation (but small strain) behaviour of the blister test specimen takes advantage of the von Karman thin plate analogy and the linear elastic fracture mechanics (LEFM) (Suo and Hutchinson, 1990). Since the thickness h_1 is assumed to be smaller than the crack extension and radius of crack front curvature, therefore the plane strain conditions are locally assumed to hold along the crack front. This allows to determine the TERR by treating the upper component as a thin clamped plate loaded by the membrane stress N and bending moment M (Jensen, 1991) as follows (Suo and Hutchinson, 1990):

$$G_T = \frac{6}{\bar{E}_1 h_1^3}\left(M^2 + \frac{h_1^2 N^2}{12}\right) \tag{2}$$

and $\bar{E}_1=E_1/(1-\nu_1^2)$ under plane strain assumption. The membrane stress and bending moment are derived for the membrane limit ($h_1 \rightarrow 0$) as (Jensen, 1991):

$$N = \sqrt[3]{p^2 E_1 h_1 a^2}\,\gamma \text{ and } M = \frac{h_1}{4}\frac{\sqrt[3]{p^2 E_1 h_1 a^2}}{\sqrt{3\left(1-\nu_1^2\right)\gamma}} \tag{3}$$

Thus, the final form of the TERR is given by (Jensen, 1991)

$$G_T = \left(\frac{p^4 a^4}{E_1 h_1}\right)^{1/3}\left(\frac{1}{8\gamma} + \frac{\gamma^2\left(1-\nu_1^2\right)}{2}\right), \tag{4}$$

where γ is the function of the upper material Poisson's ratio.

A mixed mode fracture behaviour is expected at the crack tip due to the applied type of loading and crack propagation along the interface. This mixed mode behaviour can be described by the mixed mode (or phase) angle (Suo and Hutchinson, 1990), that permits to distinguish fracture modes' contributions during crack growth. The phase angle is given by (Suo and Hutchinson, 1990)

$$\tan\Psi = \frac{\mathrm{Im}\left(Kh_1^{i\varepsilon}\right)}{\mathrm{Re}\left(Kh_1^{i\varepsilon}\right)} = -\frac{\sqrt{12}M\cos\omega + h_1 N\sin\omega}{-\sqrt{12}M\sin\omega + h_1 N\cos\omega}, \tag{5}$$

where ω is the function of the two Dundurs parameters α, β and η (Suo and Hutchinson, 1990).

The total energy release rate can be also evaluated from the general crack closure concept of the energy required to close the crack along the length Δa (Irwin, 1957). For the axisymmetric crack extension the total energy release rate is as follows:

$$G_T = \frac{1}{2\pi\Delta a}\int_{a_1}^{a_2}\left\{\sigma_N(\Delta a)g_N(\Delta a) + \sigma_T(\Delta a)g_T(\Delta a)\right\}da \tag{6}$$

where $\Delta a=a_2-a_1$ is the crack extension. The energy release rate formula can be discretised in the finite element method (FEM) sense and numerically evaluated from nodal forces and displacements using the non-singular discretisation (no quarter point elements) around the crack tip (Raju, 1988). For that purpose, a simplified axisymmetric FEM model of the two layer composite is built using the eight node second-order quadrilateral finite elements PLANE82 implemented in ANSYS (ANSYS, 2002). These elements are used to discretise the domain of the upper component, while the lower component is replaced by supports in the horizontal (X_1) as well as vertical directions (X_2). The pressure is applied to the upper component towards X_2, and the load area and magnitude is changed during crack growth according to information from the blister test. The TERR computed from analytical expressions, eqn (1) or (4) can be compared with those determined by the FEM with eqn. 6.

3 RESULTS AND DISCUSSION

Results of two case studies are presented herein: case 1, where the upper component is relatively stiff, and the case 2, where the upper component is compliant. Thus, two composite systems were investigated, namely the polymethylmethacrylate (PMMA)/silicon wafer and pressure sensitive adhesive tape (PSAT)/PMMA. Elastic constants and thickness of upper constituents are given in Table 1.

Table 1: Mechanical properties of the blister test specimens

Component	**E [MPa]**	ν	**h_1 [mm]**
PSAT	1835.5	0.35	0.03
PMMA	2200.0	0.33	0.25

Experimental results obtained from the blister test, where the critical pressure was determined as a function of the crack length, are used to determine the R-curves.

3.1 Stiff constituent composite laminate

The pressure-crack length relationship was determined from three separate experimental tests, that corresponded to three different crack lengths. The two component specimen was loaded up to the critical pressure p_{cr} at which unstable crack growth occurred. The results showed a decreasing tendency of statistically different critical pressure results along the crack length. Since small central deflections were obtained from the FEM analysis for three crack lengths $a=1.85\times10^{-3}$, 2.85×10^{-3} and 3.85×10^{-3}m, eqn (1) was used to compute the TERR. These results are shown in Table 2 in comparison with results obtained from the FEM (eqn (6)).

Table 2: Critical total energy release rate – small deformations

Crack length [m]$\times10^{-3}$	**Analytical [J/m^2]**	**FEM [J/m^2]**
1.85	0.008	0.010
2.85	0.018	0.023
3.85	0.021	0.024

The comparison of the critical TERR shows satisfactorily agreement between results obtained from these two different approaches. The critical TERR for the smallest crack length is smaller of the order of 50% than for larger crack lengths in both cases. For linear elastic materials the R-curves are usually constant (Broek, 1986), since there is no inelastic zone or this zone is small and constant during the crack growth. Thus, results for the smallest crack length can be questionable, and the interface fracture toughness might correspond to the critical TERR for larger crack lengths.

3.2 Compliant constituent composite laminate

The pressure-crack length curves were determined from single tests. The critical pressure values were recorded at particular crack lengths (a=1.5, 2.5, 3.5, 4.5 and 5.5mm) during the stable interface fracture process. It is noted that a=1.5mm corresponds to the radius of the hole of the blister specimen. The crack growth was stable and nearly circular up to the last specified crack length. Then, the fracture instability and thereby the composite failure occurred at the last crack length (close to the specimen edge). The test was carried out for three different loading rates υ=0.0125, 0.006, 0.003mm/s. The critical pressure p_{cr} was largely influenced by the non-linear properties of the interface, such that p_{cr} increased along with increasing load rate υ.

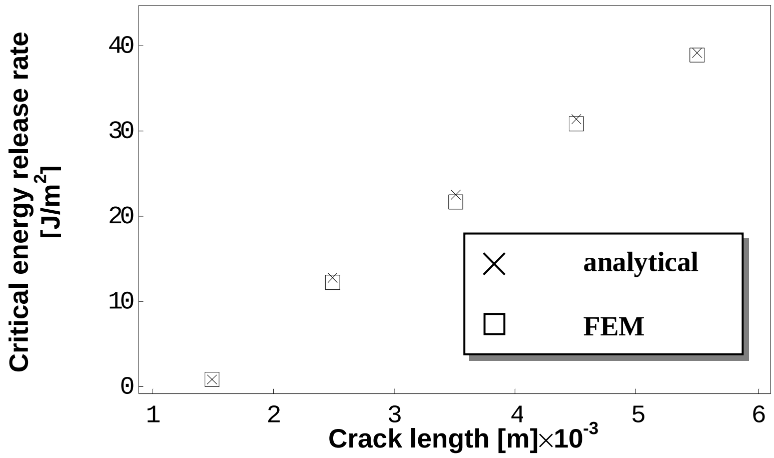

Figure 2: R-curve for υ=0.006mm/s

Both theories, geometrically linear and non-linear, were applied to compute R-curves for υ=0.006mm/s. The critical TERR determined from eqn (1) were unrealistically large and they were rejected from the analysis. Realistic values of the critical TERR were obtained from eqn (4), and compared with the critical TERR determined using the FEM (eqn (6)). The R-curves are shown in Fig. 2, where results' agreement is nearly excellent. An interesting feature of the R-curve is that it has an increasing tendency along with the crack length. Generally, the main reason for an increasing tendency of R-curve is some non-linear irreversible behaviour ahead of the crack tip, which is the function of the crack length (Broek, 1986). It is supposed that some micro-mechanisms occurring in the non-linear adhesive layer can be responsible for these effects. It is expected that as the crack propagates, the bridging or cohesive zone may increase and thereby lead to an increase of the critical TERR along with the crack length.

The mixed mode behaviour during crack propagation was revealed by the computation of the phase angle Ψ from eqn (5). Mixed mode angle Ψ was determined for ω=53.8 linearly interpolated using tables reported in Suo and Hutchinson (1990) for α=-0.3 and β=-0.09, and η=0. This phase angle was equal to Ψ=-56.6375deg and constant along the entire crack propagation range. This outcome points out, that the mode 2 prevails against the mode 1 of the TERR during the entire interface fracture process in the analysed composite.

4 CONCLUSIONS

The interface crack propagation in two layer composite materials was numerically analysed under mixed mode loading conditions. New results for the R-curve and mixed mode fracture behaviour were obtained for two different composite systems: (1) stiff/stiff and (2) compliant/stiff. The total energy release rate was numerically determined on the basis of two elasticity theories: small and large deformations theories. . The TERR values obtained from analytical expressions were in a very good agreement with FEM results. The R-curve determined on the basis of the calculated TERR and the blister test results for the stiff/stiff composite arrangement was nearly constant for larger crack lengths, while it has an increasing tendency for compliant/stiff composite material. Some bridging effects occurring at the interface are supposed to be responsible for increasing character of the curve. The phase angle value was constant during interface crack growth and the mode 2 of the TERR prevailed against the mode 1 during crack propagation in both composite laminate systems. Future work is focused on the non-linear behaviour of the compliant/stiff composite material.

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