

ADVANCED SIMULATION OF 3D FATIGUE CRACK GROWTH BY A PREDICTOR – CORRECTOR PROCEDURE CONSIDERING 3D SINGULARITIES

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ABSTRACT

An advanced incremental algorithm for the simulation of 3D fatigue crack growth in the framework of linear elastic fracture mechanics is addressed to this paper. The simulation is based on an accurate stress analysis, which is performed by the 3D dual boundary element method (Dual BEM). The utilized fracture mechanics parameters are the stress intensity factors (SIFs) and the so-called T-stresses. They are extracted from the numerical crack near-field by an optimized extrapolation and regression algorithm providing very precise values of both parameters. To answer the still open question concerning a reliable 3D crack growth criterion a predictor-corrector procedure is implemented. This concept is capable of mode-I and three-dimensional mixed-mode conditions. As part of the proposed scheme, 3D corner singularities are considered. They appear especially at the intersection of the crack front and the free surface. Based on these known asymptotic exponents, it is advantageous that the crack front angle is adjusted at such intersection points ensuring a valid square-root singularity and therefore a bounded energy release rate. As a result, the shape of the numerical crack front matches experimentally observed crack fronts locally. After applying a certain number of corrector steps assuming a constant distribution of the energy release rate along the whole crack front a global matching occurs. Finally, a sophisticated re-meshing algorithm updates the numerical model automatically for the next increment.

1 INTRODUCTION

The simulation of three-dimensional cyclic crack growth is investigated. Beside crack retardation or the process of unstable growth, the stage of stable fatigue crack growth under proportional mixed-mode loading is focused. Arbitrary crack geometries in complex three-dimensional structures with linear elastic material behavior are considered. Finally, the aim of our research is the identification of a suitable 3D crack growth criterion in combination with reliable experiments.

Therefore, it is absolutely required to cancel all assumptions and simplifications of two-dimensional models. Hence, the crack tip turns into the crack front and the crack itself is represented by an arbitrarily shaped crack surface. But, this switch introduces additional problems which are not considered in 2D and represent a real challenge to every numerical tool. Now, a complex topology and a three-axial state of stresses varying along the crack front have to be taken into account. Mixed-mode conditions in 3D imply a dependence on all three fracture modes. Moreover, the problem of intersecting crack fronts with the free surface is included. These intersection points are usually correlated to a change of 3D singularities, the so-called wedge and corner singularities.

Due to the non-linearity of crack growth an incremental procedure is required for the simulation of this phenomenon. Within each increment the following steps have to be performed. It starts with the solution of the three-dimensional boundary value problem, followed by the actual automatic 3D crack growth algorithm. This algorithm includes the determination of the new crack geometry based on a reliable crack growth criterion. Then, the numerical model is updated by a sophisticated re-meshing algorithm in a fully automatic way before the next increment is going to be proceeded.

The three-dimensional boundary value problem is solved by a special formulation of the BEM in case of cracked samples – the so-called Dual Discontinuity Method (DDM), see Partheymüller

et al. [1]. This method is a powerful tool for linear-elastic stress concentration problems. Especially in the area of high stress concentrations – ahead of the crack front – no discretization is needed. As the crack propagates into this region without any discretization, the update of the numerical model is much easier compared to volume oriented methods.

For more information concerning the stress analysis, the reader is referred e.g. to Partheymüller et al. [1] or Kuhn and Kolk [2]. The basic principle of the 3D crack growth algorithm is explained in section 2 and the predictor-corrector scheme in section 4.

2 ADVANCED 3D CRACK GROWTH ALGORITHM

The 3D crack growth algorithm starts directly after solving the boundary value problem of the current crack configuration. Then, three steps are performed to realize the incremental crack growth.

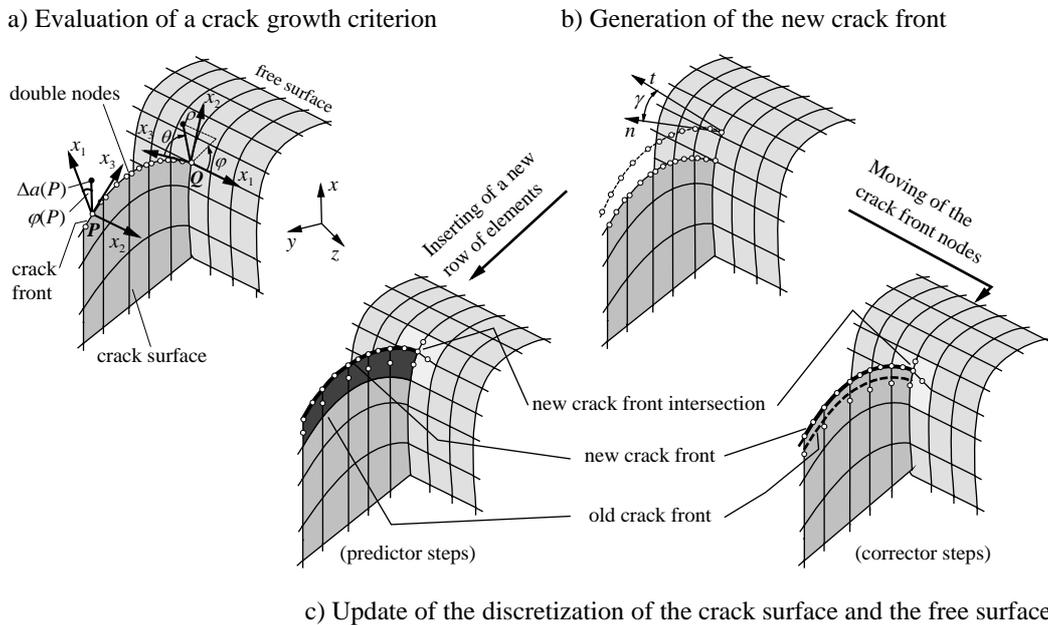


Figure 1: Basic principle of the 3D crack growth algorithm.

Firstly, relevant fracture mechanics parameters – the classical SIFs and the T-stresses – are evaluated along the whole crack front. It even requires a 3D singularity analysis, c.f. section 3. Additionally, the energy release rate is determined as a function of all SIFs, which holds in terms of linear elasticity. Based on these parameters, a suitable crack growth criterion is utilized to specify local crack extensions $\Delta a(P)$ and kink angles $\varphi(P)$ at discrete points P . This is realized by the proposed predictor corrector procedure.

Secondly, these two quantities define the shape and position of the new crack front discretely. Hence, the new crack front is generated by a cubical smoothing spline, Bronstein et al. [3], and approximated via piecewise quadratic shape functions.

Thirdly, the discretization is variably updated depending on the amount of the crack extension. If there are significant crack extensions along the whole crack front a new row of elements is inserted as indicated in figure 1c. This is typically done during a predictor step. In case of corrector steps only small local crack extensions occur. Then, the old crack front nodes are moved

towards the location of the new crack front. But, special attention is needed in case of surface breaking cracks. On one hand, the free surface has to be considered in the process of updating the discretization. To ensure an optimized mesh a sophisticated automatic local re-meshing procedure is applied. On the other hand, the 3D singularities in the vicinity of the intersection of the crack front and the free surface may change if the crack is growing. Therefore, a permanent monitoring of these singularities is necessary to be able to adjust the crack front angle γ in every increment appropriately.

3 CLASSICAL AND GENERALIZED STRESS INTENSITY FACTORS

It was indicated in the last section that the crack growth algorithm is mainly based on the evaluation of the classical SIFs along the whole crack front. As a matter of fact these SIFs are linked to the well-known square-root singularity. But this kind of singularity doesn't hold for non-smooth parts of the crack front. In contrast to 2D, in 3D it has to be distinguished between two different types of singularities, the wedge and corner singularities. Therefore, the issue of stress intensity factors needs to be investigated in more detail.

At smooth parts of the crack front the so-called wedge singularity is present. In homogeneous linear elastic material this singularity corresponds to the square-root singularity with a multiplicity of three. The asymptotic distribution of the stresses in the crack near-field is given by

$$\sigma_{ij}(r, \varphi, P) = \sum_{M=I}^{III} \frac{K_M(P)}{\sqrt{2\pi r}} f_{ij}^M(\varphi) + T_{ij}(P) + O(\sqrt{r}) \quad (1)$$

with respect to a local cartesian crack front co-ordinate system including an associated polar co-ordinate system in the (x_1, x_2) – plane, see figure 1a. Eqn. (1) holds for arbitrary points P along smooth crack fronts, c.f. Leblond [4]. The classical SIFs $K_M(P)$ with $M=(I, II, III)$ characterize the intensity of the square-root singularity and may change along the crack front. $f_{ij}^M(\varphi)$ denote the angular functions and $T_{ij}(P)$ the non-singular terms – the so-called elastic T -stresses. Both fracture mechanics parameters ($K_M(P)$ and $T_{ij}(P)$) are very accurately calculated from the numerical stresses in the crack near-field via an optimized local extrapolation method based on a regression analysis, see Kolk and Kuhn [5].

At non-smooth parts of the crack front and especially at the intersection of the crack front and the free surface 3D corner singularities have to be taken into account. In principle, they are not a-priori known and depend on the geometry in the vicinity of the singular point Q and on the elastic material parameters. The stress field in the neighborhood of such points

$$\sigma_{ij}(\rho, \theta, \varphi, Q) = \sum_{L=1}^{N_0} K_L^*(Q) \rho^{\alpha_L - 1} g_{ij}^L(\theta, \varphi, Q) + T_{ij}^*(Q) + O(\rho^{\alpha_{N_0+1} - 1}) \quad (2)$$

is asymptotically expanded with respect to a spherical co-ordinate system (ρ, θ, φ) as indicated in figure 1a and mainly characterized by the asymptotic exponents α_L . The intensities of the respective singularities are given by generalized stress intensity factors $K_L^*(Q)$. The asymptotic exponents α_L are extracted by the solution of a quadratic eigenvalue problem in terms of α , Dimitrov et al. [6] or Kolk et al. [7]. The corresponding eigenvectors are obtained simultaneously and represent the angular functions g_{ij}^L . The lower bound of α_L is -0.5 to guarantee a bounded elastic energy, c.f. Dimitrov et al. [6]. The interval $-0.5 < \alpha_L < 1$ ($L=1, 2, \dots, N_0$) is considered because the asymptotical behavior excluding the known rigid body motion modes ($\alpha=0$ and $\alpha=1$) is focused.

The classical SIF-concept only fails at some special points with α_L not equal to 0.5. But to be still able to apply this concept for the description of the behavior in the crack near-field the SIFs are numerically defined at these particular points, c.f. Nakamura et al. [8]. If α_L is greater than 0.5

and less than 1.0 the stresses are still singular but weaker compared to the square-root singularity. Hence, $K_M(P)$ tends to zero as P tends to Q . If α_L is less than 0.5, $K_M(P)$ tends to infinity. Therefore, the asymptotic exponents have to be known to determine even the classical SIF asymptotically.

4 PREDICTOR-CORRECTOR PROCEDURE

An appropriate 3D crack growth criterion is required to trace experimentally observed crack fronts as realistic as possible. Such a criterion usually depends on a user-defined incremental length Δa_0 or a pre-defined number of load cycles N . But the new crack profile may differ from experimental crack fronts depending on the choice of Δa_0 or N . Therefore, a predictor-corrector procedure is required.

The proposed scheme is based on the energy release rate, respectively the SIFs, and includes two assumptions. Firstly, the square-root singularity holds along the whole crack front, even in the vicinity of the crack front and the free surface. This is ensured by a 3D singularity analysis at this intersection as mentioned in the last section, followed by a suitable adjustment of the corresponding crack front angle. Secondly, it is assumed that the shape of the crack front corresponds to a constantly distributed energy release rate.

In case of mixed-mode, both quantities, the crack extension and the kink angle, have to be predicted and corrected. They will be explained in the order they are evaluated.

4.1 Crack extension

The crack extension is immediately known from a user-defined incremental length Δa_0 in 2D. But in 3D, this length has to be distributed along the whole crack front. This is done according to the local state of stresses expressed in terms of relevant fracture mechanics parameters (the energy release rate G or an effective SIF K_I). The maximum crack extension is either assigned to a maximum or mean value of these parameters and linearly distributed along the crack front. Alternatively, an exponential distribution, e.g. based on the Paris-Law, is applied.

Because the energy release rate has the most physical meaning, the crack extension is performed on the basis of this energy with the following distribution

$$\Delta a(P) = \Delta a_0 \frac{G(P) - C \cdot G_{\min}}{G_{\max}} . \quad (3)$$

The *corrector* C conducts the switch between predictor and corrector steps.

If C equals 0, the *predictor* is turned on. The influence of the amount of Δa_0 on the shape of the crack front and the resulting crack path can be easily verified. If Δa_0 is too small, a lot of increments are needed to find the correct shape. The drawback is a rather CPU and time intensive crack growth simulation. If Δa_0 is too large, it is difficult to approach the correct shape of experimentally observed crack fronts. Therefore, corrector steps are introduced by setting the *corrector* C equal to one, to improve the shape of the numerically determined crack fronts.

If the *corrector* is turned on ($C=1$), corrector steps are performed within an iterative procedure to ensure a constant distribution of G along the whole crack front. This assumption corresponds to a constantly distributed SIF K_I in the special case of mode-I. Moreover, based on preliminary tests it is mostly verified that the energy release rate controls the crack growth and the crack front will propagate in a way that G is constant along the whole crack front. During the corrector steps there is no crack growth in that point where the energy release rate is minimal. The crack front is only locally corrected in the direction of the propagating crack relatively to the point having a minimal G . Hence, the scheme can be named "Iterative Forward Predictor-Corrector Procedure". The numerical constancy will be obtained after several corrector steps.

To evaluate the energy release rate – as a function of all SIFs – in the framework of linear elasticity it is essential that the square root singularity holds along the whole crack front, even at both ends in case of surface breaking cracks. Knowing the present value of the asymptotic exponents α_L , the crack front angle γ of the new crack front has to be adjusted. γ is defined between the tangent of the crack front and the inward directed normal vector of the free surface. The square-root singularity is satisfied by the condition $\alpha_L = 0.5$. But, which exponent out of N_0 different exponents should be used to modify the crack front angle?

To answer this question, it has to be noticed that it cannot be distinguished between pure mode-I,II,III any more at such points with a 3D corner singularity. Only a distinction between symmetric and anti-symmetric modes is possible. The symmetric mode corresponds to mode-I and the anti-symmetric modes can be either pure mode-II or pure mode-III or a combination of both. Depending on the orientation of the crack and the boundary conditions different modes and consequently different exponents α_L are activated.

In case of a symmetric crack opening (mode-I) the crack front angle has to be adjusted that the single exponent α_L^I related to the symmetric opening mode equals 0.5. All remaining exponents are not activated. A first agreement that the special crack front angle belongs to a valid square-root singularity, even in the vicinity of the crack front intersection, can be found in Bazant et al. [9]. Detailed recent experimental investigations regarding different cross sections and crack front shapes yielding the same correlation are presented by Heyder et al. [10].

But in case of mixed-mode different exponents α_L ($L=1, \dots, N_0$) are activated. It is a reasonable choice to use the smallest activated one to adjust the crack front angle γ ensuring a crack front with a bounded energy release rate. If the smallest term α_L^I tends to 0.5, all remaining exponents are shifted to a certain value greater or equal to 0.5. This means, all SIFs are finite and the energy release rate is also finite and therefore bounded. A first coincidence to experimental results including observed crack front angles is shown by Dimitrov et al. [11].

4.2 Kink angle

If the crack extension $\Delta a(P)$ is known at discrete points along the crack front, the corresponding kink angle $\varphi(P)$ is calculated. The popular maximum tangential stress (MTS)-criterion provides $\varphi^{\text{MTS}}(P)$ only as function of K_I and K_{II} . As K_{III} appears under three-dimensional mixed-mode conditions it is obvious to consider this term, too. As the MTS-criterion only describes a tangent to a possibly deflecting crack path, a “zigzag”-path around the smooth curved path may occur. Moreover, the crack extension $\Delta a(P)$ is finite in the incremental crack growth procedure, especially in a predictor step, and not infinitesimal for which the angle $\varphi^{\text{MTS}}(P)$ originally holds.

The missing SIF K_{III} can be included following the proposition of Fulland et al. [12], that the crack growth angle is perpendicular to the maximum principle stress on an imaginary cylindrical sphere around the crack front. This improvement of the MTS-criterion can be seen in figure 2a and the modified angle is called $\varphi^{3D}(P)$. Unfortunately, there is still a tangent to a possibly deflecting crack path described. The elimination of this last drawback is part of the corrector step.

The correction of the kink angle is performed by considering higher order terms in the expansion of the stresses in the crack near-field. Namely, the T -stresses are included. It gives a kink angle $\varphi_{\text{corr}}^{3D}(P)$ depending on all SIFs (K_I , K_{II} and K_{III}) and all T -stresses (T_{11} , T_{33} and T_{13}). This angle additionally depends on the Poisson's ratio ν and on the radius r which belongs to the polar coordinates in the local crack front co-ordinate system. Particularly, the consideration of this radius includes the missing crack deflection. With $r = \Delta a(P)$, the kink angle is automatically corrected following the crack deflection closely.

The distribution of $\varphi_{\text{corr}}^{3D}(P)$ is plotted in figure 2b. It shows the kink angle in the special case of $K_I / |K_{II}| = 1$ as function of K_{III} and the T -stress with the most influence on this angle - T_{11} . As it can

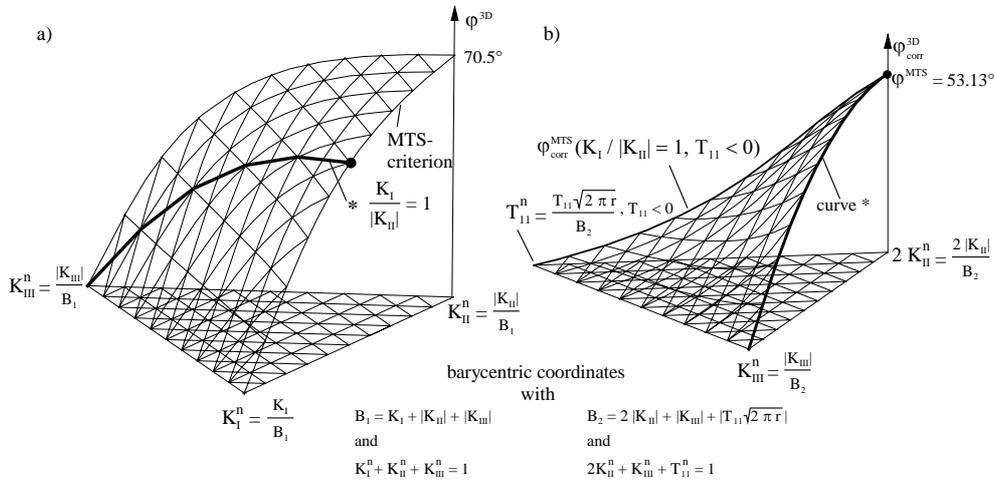


Figure 2: Kink angle depending on all SIFs and a negative T_{11} .

be seen, beside K_{III} a negative T_{11} leads to a further decreasing kink angle depending on the radius r . Concluding, the MTS-criterion and the ϕ^{3D} -criterion in each case with and without corrections by T_{11} are linked by this diagram.

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