LAP JOINT MSD ASSESSMENT USING A PROBABILISTIC MODEL

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ABSTRACT
Multiple Site Damage (MSD) is one of the major threats to airworthiness of ageing aircraft. To avoid this threat, recent recommendations by regulators stipulate an inspection starting point (ISP) and a structural modification point (SMP) in the service life of aircraft. Therefore, the capability to calculate service life to MSD onset becomes of considerable importance. In this work, a probabilistic model for prediction of MSD onset, considering both fatigue crack initiation and crack propagation as random variables, is presented and the ISP and SMP are determined for a riveted lap joint. A formulation for coupling the dual boundary element method with Monte Carlo simulation is proposed for representing the probabilistic nature of crack growth. The results obtained from the model are critically discussed by comparison with experimental data. It is noticed that the Monte Carlo simulation results were able to enclose both fatigue crack initiation and fatigue crack propagation scatter bands compared to experimental work from the literature, demonstrating the effectiveness of the model for MSD assessment.

1 INTRODUCTION

Large passenger aircraft when kept in service for an extended period of time suffer from the development of a range of damage processes associated with ageing aircraft. These can take the form of corrosion, together with various forms of fatigue failure. Multiple Site Damage (MSD) is one of the major threats to airworthiness of such ageing aircraft. MSD is the simultaneous development of fatigue cracks at an array of similar structural details. MSD has been most apparent in fuselage lap joint structures, and can result in unexpected catastrophic failure of aircraft, as it is difficult to detect. Recent recommendations by regulators to avoid this MSD threat (AAWG [1]) stipulate an inspection starting point and a structural modification point in the service life of aircraft. These points can be defined in terms of MSD analysis results, test results or by service experience. The intention is that the aircraft shall not be operated while there is a significant probability that MSD is present. Capability to accurately calculate service life to MSD onset becomes of considerable importance.

Previous workers have approached this problem by considering the probabilistic nature of MSD occurrence, and have employed Monte Carlo techniques to simulate the stochastic nature of fatigue crack initiation at fastener holes and /or subsequent crack propagation, and therefore calculate the distribution of lives to MSD onset, link-up and ultimate failure.

The crack initiation stage is commonly addressed by applying Monte Carlo simulation to lognormal or Weibull distributions of lives to achieve a specified crack size a_o (AAWG [1]). The following crack propagation stage is simulated either deterministically or probabilistically. There are particular difficulties in calculation of stress intensities for crack growth in MSD crack configurations because the β correction term will change for every different crack configuration simulated. Therefore the technique used for stress intensity calculation must be accurate and economical of computer time if it is to be used in a repeated simulation such as the Monte Carlo. In previous work, finite elements (Santgerma [2]), alternating finite elements (Proppe [3]), boundary elements (Aliabadi [4]), dual boundary elements (Kebir [5]) and compounding method (ESDU [6]) have all been used to calculate stress intensities of MSD cracks.

Over the past 10 years, there have been several Monte Carlo simulations of the MSD life calculation problem. These have differed greatly in their built-in assumptions, approximations and their calculation techniques. In consequence the predicted life distributions have also varied from simulation to simulation, as has the level of agreement with experimental data. The dual boundary element (DBE) technique for calculation of stress intensities in MSD situations has advantages in accuracy over other numerical techniques (Salgado [7]). Previous MSD simulations using DBE have used deterministic crack growth together with open hole geometries in their analysis (Kebir [5]). In this work the DBE method has been applied to a row of pin loaded holes to perform probabilistic crack growth simulation of MSD using the Monte Carlo approach. The agreement with previously published experimental MSD data is critically assessed.

2 MSD ASSESSMENT APPROACH

In this section, the MSD assessment model developed is presented in three separate parts: fatigue crack initiation, deterministic crack growth and probabilistic crack growth. The lap joint geometry which is analysed is shown in Figure 1 and consists of 3 rows of 9 pin loaded holes. It is subjected to a uniform remote alternating tensile stress with maximum stress of 100 MPa and an R ratio (min stress/max stress) of 0.1. The sheet is 1.6 mm thick and is of clad 2024 T3. The rivet diameter (ƒ) is 4 mm, and the pitch distance (p), the inter row spacing (s) and the edge distance (e) are all equal to 20 mm. The ultimate tensile strength, proof strength and fracture toughness are 448 MPa, 331 MPa and 32 MPa $m^{1/2}$, respectively.

In the Monte Carlo simulation, the crack initiation stage and the crack propagation stage are considered separately. An initial analysis allocates initiated cracks of 1.5 mm at each fatigue critical location with a randomly selected life to achieve that crack length. This is followed by a LEFM based crack growth analysis. Final failure occurs either by exceedance of the material fracture toughness or net section yield. Details of each process are given in the following sections.

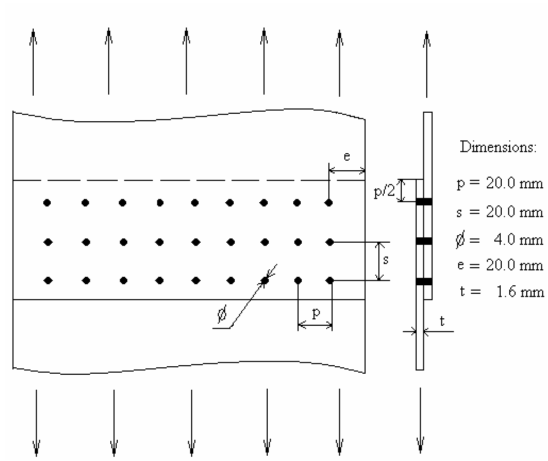

Figure 1: Lap joint configuration.

2.1. Fatigue Crack Initiation

To represent the fatigue crack initiation life 'N_0', a lognormal distribution of lives to achieve a crack size of 'a_0' is employed. Considering the external rows of a lap joint, it is assumed that each pin-loaded hole has two fatigue critical locations (FCL) at 3 and 9 o'clock positions of the hole border. For each FCL, the normal distribution '$\log(N_0)$' is defined by the mean S-N fatigue life 'm', the standard deviation 's' and the standard normal distribution 'a' given by,

$$\log(N_0) = m + a.s \tag{1}$$

When a random value of 'a' is generated by Monte Carlo simulation, one initial damage scenario is created by attributing each FCL a different initial fatigue life given by eqn (1). The S-N fatigue curve properties used for the riveted holes is from Santgerma [2], and the values for '$m[\log]$' and '$s[\log]$' are calculated as a mean value of, respectively, 5.6370 and 0.20 for an initial crack size a_o of 1.5 mm.

2.2. Crack Propagation

The DBEM formulation utilized here for stress intensity calculation was developed by Salgado [7, 8], and it has been incorporated in the DTD code for crack growth life calculation used in this work (Salgado [9]). Crack tips emanating from pin loaded fastener holes are subjected to mixed mode stress fields, and the DBE program calculates both K_I and K_{II} components. A mixed mode stress intensity range ΔK_{eff} was calculated using the Tanaka [10] expression

$$\Delta K_{eff} = \sqrt{\Delta K_I^2 + 2\Delta K_{II}^2} \tag{2}$$

The Paris equation is used to calculate the crack growth rate (da/dN), given as a function of the effective stress intensity factor (ΔK_{eff}),

$$\frac{da}{dN} = C(\Delta K_{eff})^m \tag{3}$$

Material constants C and m values are C = 6.09E-11, and m = 2.6, obtained from (Salgado [9]). Crack growth lives are then calculated in the usual way using eqn (3), with a starting crack length a_o of 1.5 mm, the initiation crack size. As cracks grow, the Swift [11] criterion is used to define link-up. After link-up with an uncracked hole, continuing damage (USAF [12]) is assumed (an initiated crack of length 0.127 mm is assumed to start from the opposite hole border to where link up took place). Final failure occurs when residual strength becomes inadequate on either material fracture toughness or net-section yield criteria.

2.3. Probabilistic Crack Propagation

In order to represent the probabilistic nature of the fatigue crack growth, the Xing [13] formulation will be used to expand the Monte Carlo simulation applied to numerical techniques, such as the DBEM. Taking the logarithm on both sides of eqn (3) it follows,

$$\log\frac{da}{dN} = \log C + m\log(\Delta K_{eff}) \tag{4}$$

To represent the stochastic nature of crack propagation, a normally distributed variable $Z \sim N(0, s_z^2)$ is added to the logarithm of the fatigue crack growth law in eqn (4),

$$\log\frac{da}{dN} = \log C + m\log(\Delta K_{eff}) + Z \tag{5}$$

Considering the properties of the standard normal distribution, the probability that a measurement will fall in a range $Z \le Z_p$ is given by $P(Z \le Z_p) = p$, and Z_p can be written as,

$$Z_p = a_p s_z \tag{6}$$

When the probability ‘ p ’ is given, a_p can be obtained from the standard normal distribution. In this work, the value of ‘m’ is assumed constant and the probabilistic character of crack growth is attributed to the constant ‘C’. For a given value of a_p [eqn (6)], the number of cycles N_f to grow a crack from an initial crack size ‘a_0’ up to a crack size ‘a_f’ is given as,

$$N_f = \frac{1}{C\exp(a_p s_z)} \int_{a_0}^{a_f} \frac{da}{(\Delta K_{eff})^m} \tag{7}$$

Based on Virkler’s [14] findings, it is assumed here that each initial damage scenario has a unique a_p value. In this work $s_z[\log] = 0.043$ has been assumed, following Proppe [3].

3 RESULTS

The results of 400 Monte Carlo simulations are presented in Figure 2, together with 6 points from the test results of Foulquier [15]. The position of the limits of the confidence regions (Press [16]) have been corrected according to Arnold [17], so that a finite number of random simulations can produce the same confidence region size as an infinite number of simulations. Convergence of Monte Carlo simulations was checked for both fatigue crack initiation and propagation lives.

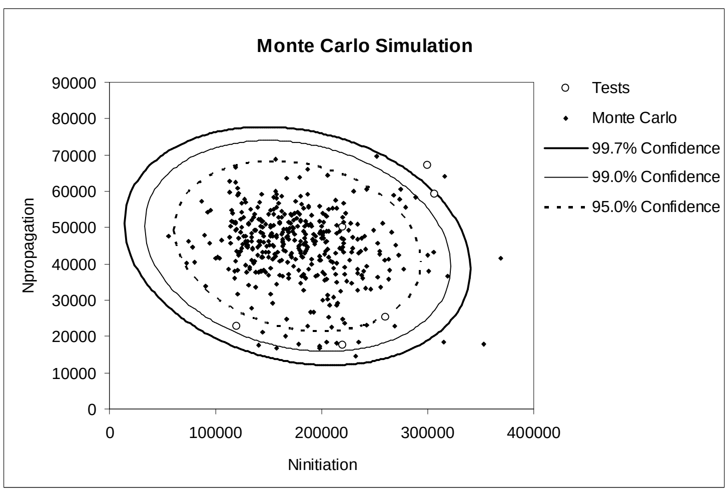

Figure 2: Monte Carlo simulation results and its confidence regions.

As found in other published simulations, for instance Santgerma [2], Proppe [3] and Kebir [5], Figure 2 shows that lives to failure are dominated by crack initiation, with initiation life occupying between 3-15 times the total propagation life. Total initiation life varies from 5.5×10^4 to 3.7×10^5 cycles, whereas propagation lives are between 1.5×10^4 to 7×10^4. Five of the six experimental points fall on or outside the 95% probability boundary line for the simulations, suggesting that real failure processes have considerably greater variability than the simulations. The mean propagation life of the experimental data is approximately the same as that of the simulations, but the spread of the 6 experimental propagation lives is as large as that of the entire 400 simulations. The spread of predicted lives encloses the range of scatter of the experimental lives for both initiation and propagation stages. However, there are only 6 experimental points;

even for the 99.7 % confidence region there is one experimental point standing outside. It is likely that were 400 experiments to be performed, the observed scatter could be significantly greater than the current data set.

4 DISCUSSION

The 6 experimental test points demonstrate a spread comparable to the 400 Monte Carlo simulation results in both Ninitiation and Npropagation axis, but most noticeably in Npropagation axis. This observation is found in the majority of previous comparisons of Monte Carlo simulation and experimental MSD data in the literature, and suggests that there are causes of scatter in propagation which are not being adequately represented in the model. For instance, Santgerma [2] has analysed the same lap joint presented in Figure 1, without considering either probabilistic crack growth or continuing damage assumption but using damage accumulation calculation for crack re-initiation; and his work gives a similar scatter band on the Npropagation axis to the one presented here. This indicates that a damage accumulation approach rather than a probabilistic crack propagation one is able to produce a similar scatter to that found in figure 2. Real MSD situations should accommodate both re-initiation and probabilistic crack growth. Simulations including both should give the desired wider scatter band.

In order to derive the Inspection Starting Point (ISP) and the Structural Modification Point (SMP), used to establish the monitoring period, the mean fatigue life to failure (Nf,mean) must be determined. From Figure 2, the value of Nf,mean is given by Nf,mean = Ninitiation,mean + Npropagation,mean = 222,000 cycles. The ISP and the SMP are calculated by dividing Nf,mean by typical factors of 3 and 2 respectively. For these numbers and for the joint analysed in this work, the ISP and the SMP values are respectively, 74,000 cycles and 111,000 cycles. Repeat inspection intervals (I_{WFD}) are established based on time from a detectable crack size initiation up to the SMP, divided by a factor (F_{WFD}). Considering the chosen initial crack size value of 1.5 mm as the detectable crack length, the total Inspection Period (IP) is defined as the cycles between the ISP and the SMP, i.e., equal to 37,000 cycles. From the 99.7 % confidence limits of Figure 2, it can be noticed that the smallest time to crack propagation ($TTCP_{MIN}$) up to failure is 12,000 cycles. According to traditional damage tolerance analysis, if $TTCP_{MIN}$ is divided by a safety factor of 2 it will lead to an inspection period of 6,000 cycles. Dividing the IP by 6,000cycles, a factor F_{WFD} = 6.2 is obtained and, consequently, a factor of 7 is more likely to be employed. Therefore, the repeat inspection intervals can be defined as $I_{WFD} = IP/F_{WFD}$ = 5,285 cycles which can be approximated to I_{WFD} = 5,200 cycles. These numbers are typical of those published in previous studies for these lap joint configurations. However the approximations and assumptions inherent in the current models, some of which are discussed above, suggest that we cannot yet regard the factors used in the derivations as fixed. It may be that distributions of real test data gathered on large numbers of aircraft would have distributions for which use of the above factors would not result in an acceptably low probability of occurrence of MSD.

5 CONCLUSIONS

(1) A probabilistic model for prediction of MSD onset considering both fatigue crack initiation and crack propagation as random variables has been presented.
(2) The dual boundary elements method has been successfully coupled with Monte Carlo simulation in order to derive a simple approach for probabilistic crack growth assessment of MSD in a riveted lap splice joint.
(3) The Monte Carlo simulation results were able to enclose both fatigue crack initiation and fatigue crack propagation scatter bands when compared to experimental work from the literature, demonstrating the effectiveness of the model for MSD assessment.

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