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DYNAMIC DELAMINATION IN THROUGH-THICKNESS REINFORCED DCB SPECIMENS

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ABSTRACT

Bridged crack models using beam theory formulation have proved to be effective in the modeling of quasistatic delamination crack growth in through thickness reinforced structures. In this paper, we model dynamic crack propagation in these structures with the beam theory formulation. Steady state crack propagation characteristics unique to the dynamic case are first identified. Dynamic crack propagation and the energetics of steady state dynamic crack growth for a Double Cantilever beam (DCB) configuration loaded with a flying wedge is examined next. We find that steady state crack growth is attainable for this loading configuration provided certain conditions are satisfied.

KEYWORDS
Dynamic, Delamination, Crack, Bridging, DCB, Stitching, Energy Release Rate

INTRODUCTION

Through thickness reinforcement of various kinds, including stitched or woven continuous fiber tows and metallic or fibrous short rods, has been developed to address the delamination problem in structural composite laminates. Substantial experimental evidence shows that through thickness reinforcement dramatically alters the delamination characteristics for the better under both static and dynamic loading conditions. For static loading, a fundamental theory based on observations of essential mechanisms is now mostly in place [1-6]. The mechanics of crack bridging by the through thickness tows have been mapped out, with governing length scales and material parameters identified [1-6]. However, equivalent fundamental knowledge and models for dynamic delamination do not exist.

This paper deals with the delamination mechanics for through thickness reinforced structures under dynamic crack propagation conditions. A beam theory formulation is adopted and certain crack propagation characteristics are identified for mode I conditions. In the next section, we examine the energetics of crack growth for a through thickness reinforced DCB specimen loaded by a flying wedge. The double cantilever beam (DCB) specimen loaded dynamically by a flying wedge offers a relatively simple experimental approach to analyzing the mode I dynamic delamination problem. Regions of stable crack growth as a function of the material properties of the through thickness reinforcement, the size of the DCB specimen and the velocity of the wedge have been identified.

Beam Theory Formulation and Solution Characteristics:
For a beam element, the equations of motion are:

$$\frac{\partial N}{\partial x} = \rho\, Bh\, \frac{\partial^2 u}{\partial t^2} \tag{1a}$$

$$\frac{\partial Q}{\partial x} - p(w,t)\,B = \rho\, Bh\, \frac{\partial^2 w}{\partial t^2} \tag{1b}$$

$$\frac{\partial M}{\partial x} - Q = \rho I \frac{\partial^2 \phi}{\partial t^2} \tag{1c}$$

where $u(x,t)$ and $w(x,t)$ are the in-plane and transverse displacements of the neutral plane respectively, $\phi(x,t)$ is the clockwise rotation of the cross-section, t is the time variable, N is the axial force, Q is the shear force, M is the bending moment, $2h$ is the total thickness of the DCB specimen, B is the width of the specimen, ρ is the density, I $(= Bh^3/12)$ is the moment of inertia and $p(w,t)$ is the bridging traction corresponding to the opening mode. In this work, the time dependent bridging traction p corresponding to the opening mode is assumed to depend only on the transverse displacement w. In the absence of an axial force N, $u = 0$.

For a Timoshenko beam, the equations for steady state motion can be reduced to [7]:

$$\begin{aligned} &\frac{\partial^4 w}{\partial X^4} + \frac{c_l^2}{(R-c_l^2)(1-c_l^2)} \frac{12R}{h^2} \frac{\partial^2 w}{\partial X^2} \\ &- \frac{1}{(R-c_l^2)} \frac{1}{Eh} \frac{\partial^2 p}{\partial X^2} + \frac{1}{(R-c_l^2)(1-c_l^2)} \frac{12R}{Eh^3}\, p = 0 \end{aligned} \tag{2a}$$

$$\frac{\partial \phi}{\partial X} = \frac{p}{REh} - \frac{(R-c_l^2)}{R} \frac{\partial^2 w}{\partial X^2} \tag{2b}$$

where $X = x - v\,t$, $c_l^2 = \rho v^2/E$, $R = \kappa G/E$, v is the (constant) steady state velocity, G and E are the shear modulus and the Young's modulus of the laminate and the dimensionless shear coefficient κ=5/6 for a beam with rectangular cross section. For steady state dynamic delamination, the velocity v is the delamination crack tip velocity.

For an Euler-Bernoulli (E-B) beam, where both shear deformation and rotational inertia are ignored, the equation for steady state motion reduces to a simple form given by:

$$\frac{\partial^4 w}{\partial X^4} + \frac{12 c_l^2}{h^2} \frac{\partial^2 w}{\partial X^2} + \frac{12}{Eh^3}\, p = 0 \tag{3}$$

Let us now consider a linear bridging law of the following type to represent the bridging action of the through thickness reinforcement:

$$p = p_0 + b_3 w \tag{4}$$

The linear law particularizes to the Dugdale law $p = p_0$ (for $\boldsymbol{b}_3 = 0$) and to the proportional linear law $p = b_3 w$ (for $p_0 = 0$). In the results that follow, we non-dimensionalize the variables by the laminate thickness h ($w \equiv h\,W$, $u \equiv h\,U$ and $X \equiv h\,x$). Thus, the transverse displacement obeys:

$$\frac{\partial^4 W}{\partial \boldsymbol{x}^4} + \boldsymbol{b}^2 \frac{\partial^2 W}{\partial \boldsymbol{x}^2} + b^2 W + d^2 = 0 \tag{5}$$

For Timoshenko beam:

$$b = \sqrt{\frac{12\ c_l^2\ R}{(1-c_l^2)(R-c_l^2)} - \frac{b_3 h}{(R-c_l^2)E}}\ ; \qquad b = \sqrt{\frac{R}{(1-c_l^2)(R-c_l^2)}\frac{12 b_3 h}{E}}\ ;$$

$$d = \sqrt{\frac{R}{(1-c_l^2)(R-c_l^2)}\frac{12 p_0}{E}}\ ; \tag{6a}$$

For Euler-Bernoulli (E-B) beam:

$$\boldsymbol{b} = \sqrt{12\, c_l^2}\ ; \qquad b = \sqrt{\frac{12\, b_3 h}{E}}\ ; \qquad d = \sqrt{\frac{12\ p_0}{E}}\ ; \tag{6b}$$

The general solution to Eqn. 5 is:

$$\begin{aligned} W(x) \;=\; & -\frac{d^2}{b^2} + K_1 e^{-\sqrt{-\frac{b^2}{2} - \frac{1}{2}\sqrt{b^4 - 4b^2}}\,x} + K_2 e^{+\sqrt{-\frac{b^2}{2} - \frac{1}{2}\sqrt{b^4 - 4b^2}}\,x} \\ & + K_3 e^{-\sqrt{-\frac{b^2}{2} + \frac{1}{2}\sqrt{b^4 - 4b^2}}\,x} + K_4 e^{+\sqrt{-\frac{b^2}{2} + \frac{1}{2}\sqrt{b^4 - 4b^2}}\,x} \end{aligned} \;. \tag{7}$$

There are three regimes to the solution behaviour that are independent of the boundary conditions, and they have been identified below (Note: $S = \boldsymbol{b}_3\, h/(12\, \boldsymbol{k} G)$):

- <u>Case 1:</u> $\beta^2 < 0$ and $\beta^4 > 4b^2$ => Exponential behavior
- For Timoshenko beam, this is true provided:

$$\frac{r\,v^2}{E} \le \frac{S}{1+S} \quad \text{and} \quad 3\,(S(1-c_l^2) - c_l^2)^2 \ge S(1-c_l^2)(R-c_l^2) \tag{8a}$$

- For the E-B beam, the above condition is never satisfied.

- <u>Case 2:</u> $\beta^2 > 0$ and $\beta^4 > 4b^2$ => Oscillatory and non-decaying behavior
- For Timoshenko beam, this is true provided:

$$\boldsymbol{r}v^2 / E > S/(1+S) \quad \text{and} \quad 3\,(S(1-c_l^2) - c_l^2)^2 \ge S(1-c_l^2)(R-c_l^2) \tag{8b.1}$$

- For the E-B beam, this condition is satisfied when: $\boldsymbol{r}v^2 / E \ge 2\sqrt{S\ R}$ (8b.2)

- <u>Case 3:</u> $\beta^4 < 4b^2$ => Oscillatory with exponential decay behavior
- For the Timoshenko beam, this is true when:

$$3\,(S(1-c_l^2) - c_l^2)^2 < S(1-c_l^2)(R-c_l^2) \tag{8c.1}$$

- For the E-B beam, this is true when: $\boldsymbol{r}v^2 / E < 2\sqrt{S\ R}$ (8c.2)

The conditions determined above give us insight into when dynamic effects can significantly alter the mechanisms of deformation and the resultant bridging phenomena. For instance, if the crack tip velocity exceeds the condition prescribed in Eqn. 8a, oscillatory displacement fields will be introduced in the wake of the crack, and these multiple oscillations could lead to crack face interpenetration. When such oscillations are present, the mechanics of bridging and the efficacy of through thickness bridging ligaments on the energetics of crack growth will be considerably altered. For example, stick-slip propagation modes would appear to be possible, as contacting fracture surfaces bounce. The complex details of such a possibility will be considered elsewhere. Here, we model the arms of the DCB specimen as an EB beam and study propagation characteristics up to the point of fracture surface contact, which is a simpler problem. (Constants β, b and d are given in Eqn.6b)

Wedge-Loaded Double Cantilever Beam

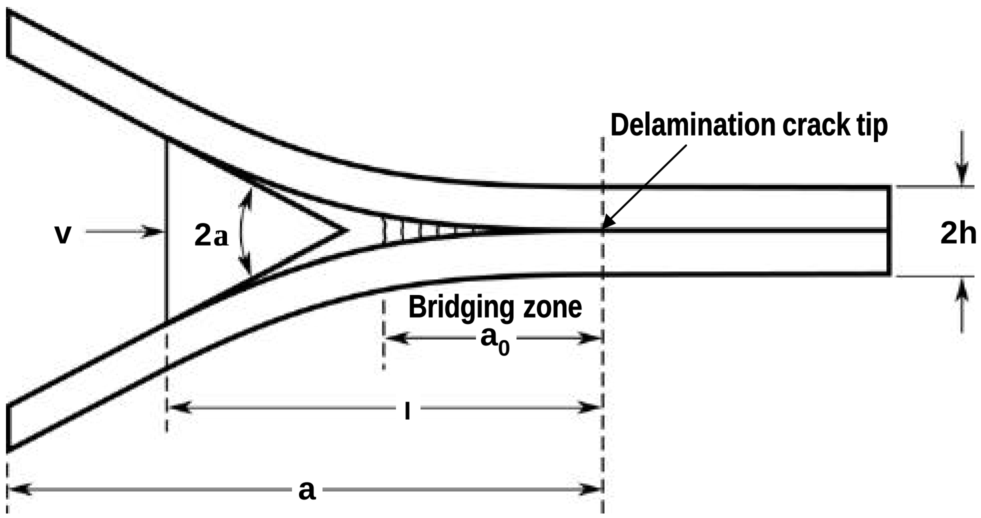

Figure 1: Schematic of through thickness reinforced DCB specimen loaded with a flying wedge

The double cantilever beam (DCB) specimen loaded dynamically by a flying wedge, of constant velocity v, offers a relatively simple experimental approach to studying the mode I dynamic delamination problem (Figure 1). The test is especially attractive for studying the bridging effects supplied by through-thickness reinforcement (e.g., stitches or rods) in laminates. In figure 1, $2\boldsymbol{a}$ is the wedge angle, l is the distance between the wedge and the crack tip and a_0 is the length of the bridging zone. In non-dimensional form, $l \equiv h\,L$, and $a_0 \equiv h\,A_0$. The role of the bridging on the crack energy release rate is determined in this section. We assume that the crack propagates under steady state conditions and confirm the possibility of steady state propagation by finding consistent solutions. Further, we assume the bridging zone size is invariant and translates with the crack tip.

For the unbridged portion, the deflection profile ($w_u \equiv h\,W_u$) is obtained by setting b = d = 0. Therefore:

$$\frac{\partial^4 W_u}{\partial \boldsymbol{x}^4} + \boldsymbol{b}^2 \frac{\partial^2 W_u}{\partial \boldsymbol{x}^2} = 0 \qquad \text{for } (-L \le x \le -A_0) \tag{9a}$$

$$\frac{\partial^4 W}{\partial x^4} + b^2 \frac{\partial^2 W}{\partial x^2} + b^2 W + d^2 = 0 \qquad \text{for } (-A_0 < x \le 0) \tag{9b}$$

The relevant boundary conditions are:

$$W(x=0)=0\,,\; W^{'}(x=0)=0\,,\; W_u^{'}(x=-L)=-a\;,\; W_u^{''}(x=-L)=0\;. \tag{10}$$

The governing Eqn. 9 together with the boundary conditions (10) and the continuity conditions at the end of the bridging zone ($x=-A_0$) will determine the deflection profile of the beam. Note that the bridging zone length (A_0) will be dictated by the critical crack opening displacement ($w_c \equiv h\,W_c$) required for failure of the bridging ligament. The crack energy release rate (G_{Total}), as determined through the total energy balance is:

$$G_{Total} = \frac{1}{B}\left(\frac{\partial U_{ext}}{\partial a} - \frac{\partial U_s}{\partial a} - \frac{\partial U_k}{\partial a}\right) \tag{11}$$

where U_{ext} is the work done by the applied load, U_s is the strain energy, U_k is the kinetic energy, B is the uniform width of the DCB specimen, and a is the crack length. For steady state crack extension $a = vt$, where v is the crack velocity and t is time. For the DCB specimen loaded with a flying wedge this reduces to:

$$G_{Total} = \frac{a\,E\,h}{6}\left.\frac{\partial^3 W_u}{\partial x^3}\right|_{x=-L} - r\,h v^2 a^2 \tag{12}$$

In addition, by application of the dynamic J-integral, the energy released at the crack tip is related to the bending moment M by [8]:

$$G_{Tip} = \frac{12}{E\,h^3}\left(M_{x=0}\right)^2 = \frac{E\,h}{12}\left(\left.\frac{\partial^2 W}{\partial x^2}\right|_{x=0}\right)^2 \tag{13}$$

Comparing G_{Total} and G_{Tip} for the displacement fields derived for the linear bridging law, one finds that

$$\Delta G_b = G_{Total} - G_{Tip} = 2\int_0^{w_c} p\;dw \tag{14}$$

where ΔG_b represents the work that must be done against the bridging ligaments along the bridged zone. This result is identical to that for the quasi-static case for small scale bridging conditions. Since there is no rate dependence to the bridging law, it is not surprising that the small scale bridging limit relationship is obeyed.

Since we limit our analysis to small scale bridging, tow failure must occur in the wake of the crack. Small scale bridging is ensured provided the displacement profile monotonically increases within the bridging zone from the crack tip and the pull-out required for tow failure is less than the maximum crack opening displacement within the bridging zone. This condition determines a criterion for the maximum allowable bridging zone length, A_{max}, which is obtained by solving $\partial W(x=-A_{max})/\partial x = 0$. Therefore, if $A_0 \le A_{max}$, then $W_c \le W_{critical}(\equiv W(x=-A_{max}))$, and hence small scale bridging condition is ensured.

Detailed calculations of the deflection profile, the crack energy release rate and the maximum allowable bridging length can be computed with the formalism presented above for both the Dugdale bridging law and the proportional bridging law. For instance, when the bridging ligaments obey the Dugdale bridging law, steady state crack propagation with small scale bridging is provided $A_0 \leq A_{max}$, where A_{max} is given by:

$$\begin{aligned} & -2\ l\ (\ Cos(\ l\ ) - Cos(\ l\ (1-\hat{A}_{max}\ ))\ ) \quad + \quad 2\ l\ D\ \hat{A}_{max}\ (\ Cos(\ l\ (1-\hat{A}_{max}\ )) - 1\ ) \\ & + D\ (\ Sin(\ l\ (1-2\hat{A}_{max}\ )) + Sin(\ l\ \hat{A}_{max}\ )\ ) + D\ (\ Sin(\ l\ \hat{A}_{max}\ ) - Sin(\ l\ )\ ) \qquad = 0 \end{aligned} \tag{15}$$

and where $\lambda = \beta L$, $D \equiv d/\lambda$, and $\hat{A}_{max} \equiv A_{max} / L$. Regions of steady state stable crack growth under small scale bridging condition can thus be deduced as a function of the material properties of the through thickness reinforcement, the size of the DCB specimen and the velocity of the wedge.

CONCLUSIONS

The dynamic delamination cracking behavior and the energetics of crack growth in through thickness double cantilever beam (DCB) specimens has been analyzed. The role of bridging by stitches or rods in dynamic crack growth was computed by solving the bridged crack problem within the framework of beam theory. For steady state crack growth conditions, different regimes of the solution behavior have been identified which would correspond to different crack propagation characteristics. Regions of steady state crack growth under small scale bridging condition can be deduced as a function of the material properties of the through thickness reinforcement, the size of the DCB specimen and the velocity of the wedge. This provides guidelines for design of experiments to probe the efficacy of bridging on improving the dynamic fracture toughness of through thickness reinforced structures.

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