# CYCLIC THERMAL LOADING OF CERAMIC MATRIX COMPOSITES: CONSTITUTIVE MODELS AND DESIGN STRATEGIES

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## ABSTRACT

In this paper simple strategies are presented for the design of ceramic matrix composite (CMC) components which are subjected to cyclic thermo-mechanical loading histories. Simple constitutive models are described for the steady cyclic response which capture the major characteristics of the material behaviour. Analytical procedures are described which evaluate the component response in the cyclic state. The approach is illustrated by analysing the classical Bree problem assuming material properties which are representative of a SiC/SiC composite. Interaction diagrams are presented which identify safe operating conditions and the extent of damage in the component.

## **KEYWORDS**

Cyclic thermo-mechanical loading, CMCs, SiC/SiC composites

## **INTRODUCTION**

Decisions during the early stages of traditional design procedures are based on a small number of material properties and the results of simple calculations. Refinement of the design is accompanied by increasingly complex analyses which are based on a more detailed description of material behaviour. This approach has been very successful, but it is characterised by a slow and expensive development which is inconsistent with the need for designers to respond to the rapid increase in the availability of new materials. When dealing with any material it is important to identify those features of the material response which are likely to dominate in a given situation and to develop constitutive laws which allow the relationship between material behaviour and component performance to be clearly identified.

In this paper we attempt to establish a design method for SiC/SiC Ceramic Matrix Composites (CMCs), which are candidate materials for use in components subjected to severe cyclic thermal loading. We develop a simple phenomenological model of material behaviour, guided by the extensive studies of material scientists on the deformation and failure mechanisms. We limit our consideration here to situations in which the loading is largely uniaxial and the fibres within the body are aligned with the principal loading direction. Approximations to the material behaviour are introduced by concentrating on the cyclic state and excluding mechanisms which are known to have little effect on the overall material behaviour. It is then not necessary to develop evolution laws for the state variables which define the material response. Here we present the

results of a set of calculations for the classical Bree problem and construct simple interactive diagrams which give a clear description of the behaviour of the component and which can be employed readily in design.

## **IDEALISED MATERIAL BEHAVIOUR**

There has been significant progress in the development of constitutive models for the mechanical behaviour of CMCs in recent years. The micromechanical processes which determine the macroscopic response are described by Zok et al [1,2] for monotonic loading and cyclic loading conditions respectively. The understanding gained from these studies has guided the development of macroscopic constitutive laws for the material behaviour. Burr et al [3,4] have examined the behaviour within a thermodynamic framework to develop a damage mechanics model which considers the different contributions to the degradation process in a consistent manner. Models of this type require extensive experimental data in order to properly calibrate them. They are useful in the final stages of design where both the material properties and component geometry need to be optimised. During the early stages of design, however, it is important to develop an understanding of the interaction between the material and structural phenomena, which combine to determine the overall structural response and to identify those features of the material behaviour which most critically influence the component performance. Here we describe a simple material model which captures the dominant features of the material response under cyclic loading histories. We are primarily interested in the material response after thousands of cycles. We assume that a cyclic state is reached for the class of loading histories of interest here and develop simple models which describe the steady cyclic response of the material.

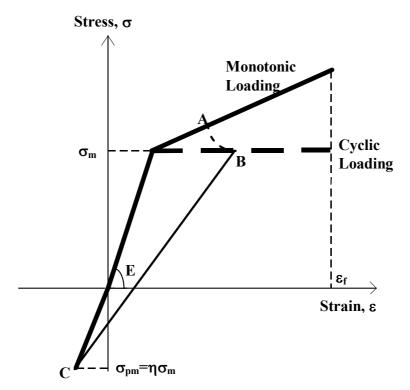
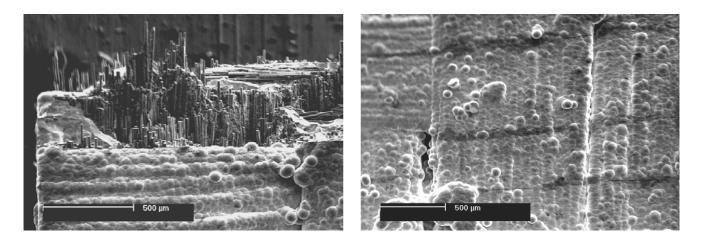


Figure 1 Idealised stress/strain response of a SiC/SiC composite

The idealised response of a SiC/SiC unidirectional composite is shown in Figure 1. During monotonic loading the response follows that indicated by the solid line in the Figure. If the stress exceeds the matrix cracking stress,  $\sigma_m$ , then during subsequent cyclic loading the inferface between the fibre and matrix gradually wears away. As a result, the compliance of the material gradually decreases and the maximum stress experienced during a cycle at a point in the component follows a trajectory similar to that illustrated

by the path AB, until the stress reaches a threshold value, below which no further deterioration of the mechanical response occurs. In situations where a component experiences a large number of cycles during its life (typically greater than 1000) a cyclic state is achieved, with the peak stress lying along the cyclic loading curve of Figure 1, where the stress is coincident with the threshold stress for strains above the matrix cracking strain. For simplicity, the threshold stress in Figure 1 is taken equal to the matrix cracking stress. This is a valid approximation for many SiC/SiC composites. Provided the strain accumulated during this process is less than a critical strain  $\varepsilon_{f}$ , damage within the body remains as discrete microcracks. If, from equilibrium considerations, the maximum stress at a point in the body is required to exceed the threshold stress the component will eventually fail. In the cyclic state, if the stress is reduced from the peak value the unloading line BC is followed until a compressive stress  $\sigma_{pm}=\eta\sigma_m$  is achieved. At this point all the microcracks are closed and further unloading follows the elastic line for the virgin material. All unloading lines pass through the same point C on this line. The magnitude of  $\sigma_{pm}$  depends on the residual stresses induced in the material during processing.

The material response represented by Figure 1 can be characterised by specifying the modulus of the virgin composite, *E*, the matrix cracking stress,  $\sigma_m$ , the crack closure stress,  $\eta\sigma_m$  and the strain to failure, which we normalise to define the quantity  $\beta = E\varepsilon_f / \sigma_m$ .



(a) (b) **Figure 2** (a) The failure surface of a SiC/SiC composite subjected to a constant axial load and cyclic thermal loading history involving through thickness temperature gradients. (b) A micrograph of the surface of the specimen within the thermally cycled region away from the failure plane, showing microcracks hich developed during the thermal loading history

## THE BREE PROBLEM

In this paper we restrict our consideration to situations in which there is a through-thickness temperature gradient in the component. There are two possible mechanisms of failure for thermal loading histories of this type. Failure can either be determined by the growth of delamination cracks [5,6], or by the growth of microcracks through the thickness of the sample [7], followed by fibre failure and pull-out. Booker [8] has recently conducted a series of tests on a number of different SiC/SiC composites in which plane specimens were subjected to a constant axial load. A small region on the surface of the sample was heated using three focused infrared lamps, while the opposite face was cooled using a chiller unit. This set-up allowed the specimen to be subjected to a cyclic thermal loading history during which the maximum temperature difference across the plate was of the order of 800°C, with a staedy state temperature variation of 450°C. A typical failed component is shown in Fig 2. In all the tests there was no evidence of delamination and failure occurred by general micracking and fibre failure. It is therefore appropriate to analyse this class of loading history using the material model described in the previous section, which is based on matrix cracking, interface degradation and fibre failure and pull-out.

In order to gain insight into the relationship between material behaviour and structural performance we use the idealised model of Figure 1 to analyse the classical Bree problem represented in Figure 3, where the plate is subjected to a constant axial stress  $\sigma_p$  and a cyclic thermal loading history, whereby one side of the plate experiences a constant temperature  $\theta_o$ , while the temperature of the opposite side is subjected to a temperature which is cycled between  $\theta_o$  and  $\theta_o + \Delta \theta$ . The temperature is assumed to be cycled sufficiently slowly so that it varies linearly across the plate througout the cycle. We can characterise the thermal loading in terms of the maximum thermo-elastic stress experienced during a cycle,  $\sigma_t = \frac{1}{2} E \alpha \Delta \theta$ , where  $\alpha$  is the linear coefficient of thermal expansion.

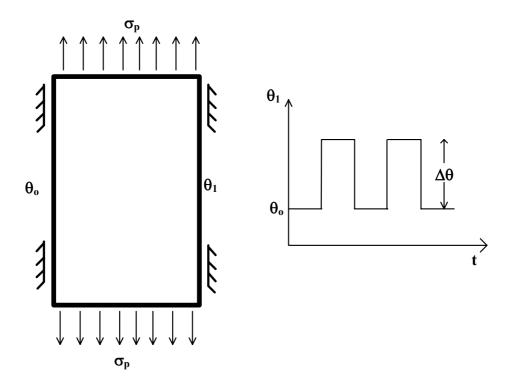


Figure 2 Classical Bree cyclic thermal loading problem

We assume that failure occurs when the strain in any part of the plate reaches the critical value  $\varepsilon_f$ . The analysis is quite lengthy, but the results can be presented in a simple graphical form. The critical strain is first achieved on the cold side of the plate when there is a temperature gradient. The combination of  $\sigma_p$  and  $\sigma_t$  which result in failure is shown as a solid curve in Fig 3 for  $\beta$ =6.93 and  $\eta$ =0.2, which are typical values for a SiC/SiC composite. The dashed line in this figure represents the combination of thermal and mechanical loading at which matrix cracking first occurs. This is often interpreted as the design limit. It is evident from this plot, however, that by taking into account the full effects of matrix cracking a much higher design limit is predicted for the component. There are three major contibutions to this large increase in load carrying capacity: the actual strain to failure is almost seven times the strain at the start of matrix cracking; the residual stress and its influence on crack closure; and most importantly, the influence of material damage on Young's modulus, which results in a decrease in the stress range experienced during cyclic thermal loading.

The analysis also provides information about the extent of microcracking in the plate. Three different regimes of behaviour can be identified as illustrated in Figure 4. In regime I, which occurs at high mechanical loads, the entire body is microcracked and the stresses are tensile throughout the cycle, such that these cracks are always open. In regime II, the entire body is microcracked, but the hot side of the plate goes into compression when there is a temperature gradient. In the remainder of the benign cracking regime only part of the plate is cracked. The chained lines of Figure 4 represent the fraction of the plate which has

experienced microcracking. The microcracked zone spreads in from the cold side of the plate. Thus there are no cracks growing in from the hot side of the plate. If the maximum temperature experienced in the microcracked zone is less than the so called "pest temperature" [9] then environmental degradation of the interface and fibres is not likely to occur, even though part of the plate is above this temperature

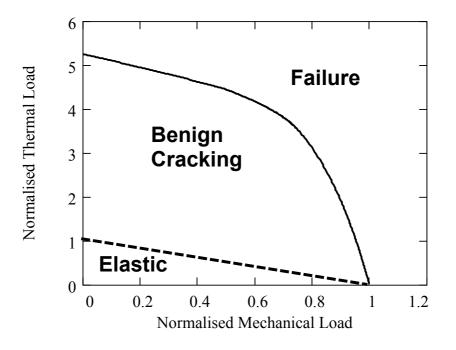


Figure 3 Interaction diagram for the problem of Fig 2

By varying the values of  $\beta$  and  $\eta$  and the magnitude of the threshold stress for fatigue damage we can examine the influence of each of these on the position of the limit boundary and the extent of microcracking in the cyclic state. For high thermal loads  $\beta$  and  $\eta$  are the most important parameters, with large values of  $\beta$  (ie large ductilities) and small values of  $\eta$  (small closure stresses) producing the best performance. Although the model of Figure 2 is a simplification of the actual response it reflects the major features of the material response and is consistent with predictions based on micromechanical models of the internal degradation processes which lead to failure. Use of these micromechanical models allows the parameters which have been identified as being important in this situation to be related back to microscopic features of the material, which can be controlled during processing to produce an optimum material for a given application. For example, a small value of  $\eta$  requires a large matrix cracking stress or small residual stresses. Calculations of this type, therefore, do not only provide valuable information to the designer, but they also provide important information to the material producer by identifying the most important macroscopic properties and the microstructural features that most strongly influence these properties.

## **CONCLUDING COMMENTS**

In this paper we have developed a simple material model for the macroscopic response of a composite material based on an understanding of the micromechanical processes which result in damage development. It is possible to relate certain features of the macroscopic stress/strain curve to the internal degradation processes and to identify which of these processes dominate and largely determine the material response. We have analysed a simple representative structural problem in which the component is subjected to a combination of thermal and mechanical loading. Significant relaxation of the thermally induced stresses occur due to the influence of material damage on the instantaneous modulus. Design calculations which take this relaxation of stress into account provide design limits which are substantially in excess of elastic procedures which do not permit the development of any matrix cracking.

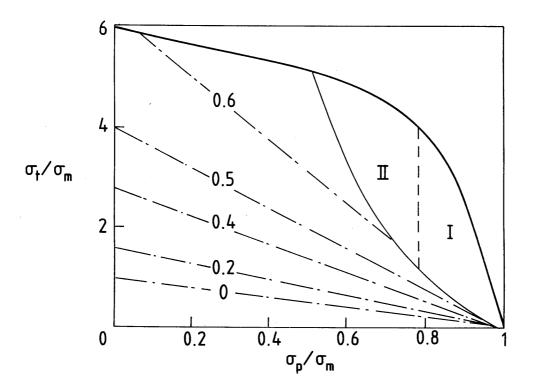


Figure 4 Interaction diagram fro the Bree problem of Figure 2. The chained lines represent the fraction of the plate which has experienced general microcracking

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