A NEW APPROACH FOR IDENTIFYING THE BENDING LOADS ON BEAM USING THE DISCRETE INTEGRAL METHOD

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ABSTRACT

Analysis of inverse problems has already been performed in various fields. In many cases, assumptions for the solution is needed. It seems that the problems which need any assumptions cause a contradiction in the analysis.

On the other hand, we have developed the discrete integral method(DIM) utilizing the delta function. We have noticed that the DIM is one of the excellent schemes to solve the inverse problem since it can solve it without any assumptions. In this paper, we attempt to apply the DIM to one-dimensional inverse problems. Namely, we developed a scheme for identifying the external load distribution on a homogeneous beam without any assumption for the solution.

Through several examples, it is proved that the present scheme gives accurate and natural solutions.

KEYWORDS

Boundary element method, Discrete integral method, Inverse problems, Bending problems of beam

INTRODUCTION

In the analysis of Inverse Problem[1,2], many points of issue have been left yet even if problems with ill-conditions are excluded. One of them is concerning presumption. This means that, it is necessary to give assumption information such as the shape, the number, its size or position and so on, of the object to be treated, as supplementary information for identification. It is difficult to solve it as an inverse problem if several assumptions concerning priori information are not defined, for the subject to be estimated is a unknown existence. In addition, the unnatural equations must be used when the number of parameters is different from that of simultaneous equations which works as a deciding condition. These cause the difficulty to establish a general scheme in the analysis of inverse problems. For these reasons, it is important to establish a scheme for inverse problems without any assumptions of the solution.

In this study, the first time, the discrete integral method utilizing the delta function is developed and we try to apply this method as one of schemes to the analysis of inverse problem by boundary element method (BEM). It is shown that the identification is performed naturally without any assumptions of the solution by using the present scheme. In this report, the present scheme is applied to one dimensional inverse problem, namely, the identification of external load distribution in bending problems of a homogeneous beam.

1. INSTITUTION OF ONE DIMENSIONAL INVERSE PROBLEM

1.1 *Integral equation of bending problem of beam by BEM*

It is convenient for the problem to identify external load distribution in bending problems of a beam to use the formulation of BEM. For a homogeneous beam under an external load of $q(x)$ as shown in Figure 1, the equation of the deflection W can be written as[3]

$$W(s) = [-WQ^* + \theta M^* - M\theta^* + QW^*]_0^L + \int_0^L q(x)W^*(x,s)dx \tag{1}$$

Where L is the length of the beam, s is the observation point, θ is the slope, M is the bending moment and Q is the share force, and parameters pointed with asterisk * are kernel functions. The inverse problem in this report is to identify the external load distribution applied on beam (i.e., $q(x)$ in the integral term of right side of Eqn.1) from the information of the deflection $W(s)$ which is monitored at observation point s as well as the information of boundary conditions at both ends of the beam.

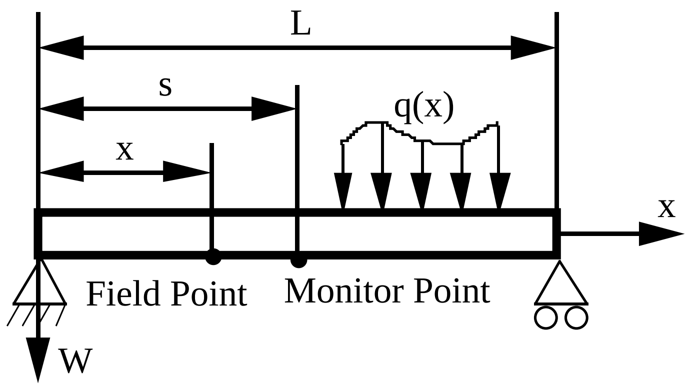


Figure 1: A homogeneous beam with external load $q(x)$

1.2 *Discrete integral method utilizing the delta function*

Here we will explain the discrete integral method utilizing the delta function, which forms the basis of this study. The following integration is considered.

$$\int_a^b f(x)K(x)dx \tag{2}$$

Here, $K(x)$ is a known function, $f(x)$ is a function which is treated as the target of interest (for example, in this case, it is the function of external load distribution), and it can be known or unknown value. The function $f(x)$ is usually approximated using the quadratic element. Instead of this, we approximate it using Dirac's delta function:

$$\nabla^n f(x) = \sum_{i=1}^{m} P_i \delta(x - x_i) \tag{3}$$

where ∇^n is the $n-$th Nabla differential operator, P_i is the strength of the delta function, namely, the strength of the virtual concentrated source, x_i is its applied position and m is the number of P_i.

Further, a function Z^* defined as the following equation is introduced:

$$\nabla^k Z^* = K^*(x) \tag{4}$$

It is supposed that the function Z^* is obtained by analytical operation. By substituting Eqn.4 into Eqn.2, and by integrating it by part, the Eqn.2 can finally be written as the following equations. Namely, in the case of $n = 2$,

$$\int_a^b f(x)K(x)dx = \int_a^b f(x)\nabla^2 Z^* dx = [f\nabla Z^* - \nabla f Z^*]_0^L + \sum_{i=1}^{m} Z^*(x_i)P_i \tag{5}$$

and in the case of $n = 4$, it becomes

$$\begin{aligned}\int_a^b f(x)K(x)dx &= \int_a^b f(x)\nabla^4 Z^* dx \\ &= [f\nabla^3 Z^* - \nabla f \nabla^2 Z^* + \nabla^2 f \nabla Z^* - \nabla^3 f Z^*]_0^L + \sum_{i=1}^{m} Z^*(x_i)P_i \end{aligned}\tag{6}$$

We found that the given integration is expressed by the quantities at the both ends of the beam and the strength of the delta function P_i, so the domain integral operation is never needed.

This scheme is regarded as a new discrete integral method, and will be introduced into the equation of beam which is expressed by Eqn.1.

1.3 *Construction of simultaneous equations for inverse problem*

Following equations are obtained when the Eqn.6 is substituted into the second term on the right side of Eqn.1 with $f(x) = q(x)$ and $K(x) = W^*(x, s)$:

$$\begin{aligned}W(s) = &[-WQ^* + \theta M^* - M\theta^* + QW^*]_0^L \\ &+ [q\nabla^3 Z^* - \nabla q \nabla^2 Z^* + \nabla^2 q \nabla Z^* - \nabla^3 q Z^*]_0^L + \sum_{i=1}^{m} Z^*(x_i)P_i\end{aligned}\tag{7}$$

In Eqn.7, the equation of BEM which gives the deflection is expressed using the strength of delta function P_i (unknown quantity). In addition to P_i, there are still the boundary values of physical quantities (the 4 parameters of W, θ, M, Q exist at each end of the beam, so the total amount is 8, and 4 of them are given by boundary conditions). Besides, the load distribution q and its differential quantity at both ends of the beam remain as unknown in the equations. To sum up, the total amount of the unknown values is $m+8$. To match with the number of the unknowns, m equations are obtained by monitoring the information of deflection at the m points where the delta function is applied. Further, 4 fundamental equations[4] to solve the bending problem of beam as a direct problem are used. For the rest 4 equations, the self-interpolated equation of q

$$q(s) = [q\nabla Y^* - \nabla q Y^*]_0^L + \sum_{i=1}^{m} P_i Y^*(x_i, s), (n = 2) \tag{8}$$

and its differential form are available. Therefore, the simultaneous equations, each has a natural form, can be constructed with the necessary number. When the simultaneous equations are solved, P_i and each unknown quantity at both ends can be calculated, then, we can calculate the external load distribution at any point of the beam using Eqn.8 directly because there is no unknown value on the right side of the equation.

2. EXAMPLES

In this section, analysis examples for model calculation are instituted, and the above identification method of external load distribution will be verified. The examples are calculated and showed by the following rules unless a special description is made:

1. The units: the dimensionless quantities are considered to fit any system of units. The length of beam L is 10.

2. The boundary condition(B.C.): a simply supported beam is treated.
3. The number of points m where the delta function is applied is 49 and they are arranged at even intervals except the both ends.
4. The order n of Nabla-operator: the equation of $n = 2$ is used.
5. In all graphs, the horizontal axis is in the length direction of beam (x axis), the vertical axis is the value of load. And, "Exact" means exact distribution, "Present" means present distribution of identified results. The rigidity EI is 100 in all examples.

Other boundary condition and the case for a Nabla-operator of order $n = 4$ will be discussed at the last.

2.1 *Identifying a concentrated load*

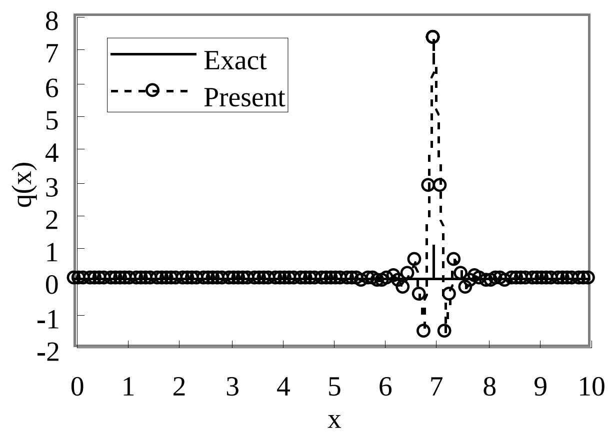


Figure 2: Identification of a concentrated load

First, the problem of a concentrated load is treated. The value of the concentrated load is 1 and it is applied at the point of $x = 7$. The result is shown in Figure 2. From this figure, it is seen that a sharp peak appears at the point of $x = 7$ and large overshoots exist near the peak. From this scene, we can ensure that a concentrated load exists there. However, the peak value differs very much from the exact value of 1. This occurs because the obtained value of q is expressed as a distributed load. Therefore, it is necessary to take attention that $q(s)$ must be integrated to identify the magnitude of the concentrated load. (This means that it is essentially impossible to distinguish a concentrated load from a distributed one in a very narrow range by only once identification using this analysis scheme. But in most cases, the integration will not be needed because practically most loads have a definite distributed range and therefore can be regarded as a distributed one. However, it will not be mentioned further.) From the above, the value of concentrated load (which may be the resultant force of distributed load in a very narrow range) is decided by the following equation in this analysis scheme:

$$F = \int_a^b q(\xi) d\xi \tag{9}$$

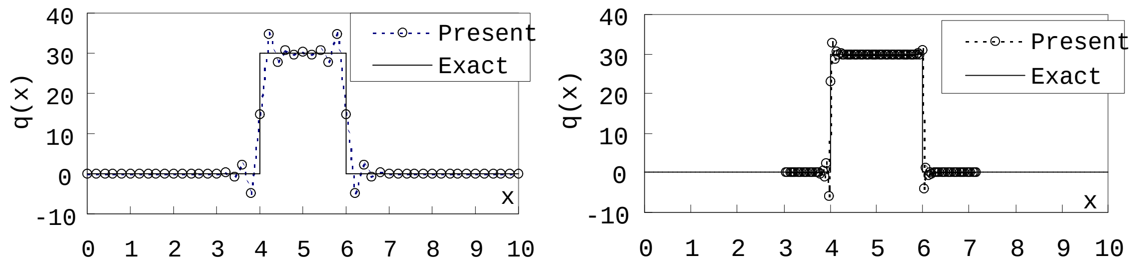
This equation can be integrated easily by Eqn.8, and the concrete expression can be obtained.

Table 1: Calculated load magnitude (Exact is 1)

Integration range	Integrated value
$6.8 \leq x \leq 7.2$	1.1423
$6.6 \leq x \leq 7.4$	0.9511
$6.4 \leq x \leq 7.6$	1.0205
$0.0 \leq x \leq 10.0$	0.9999

The magnitude of the concentrated load which was calculated by Eqn.9 for result of Figure 2 is shown in Table 1. Though the error is a little large in the range disturbed by overshoots, yet it is good enough as a estimated value. And, if we integrate it throughout the whole range, the value should become 1 because of the equilibrium condition of the force, as shown in the Table 1, so the extremely accurate value can be obtained.

2.2 *Identifying distributed load and its re-identifying*



(a) Primary identification (b) Secondary identification

Figure 3: Refined identification of a localized quadratic distributed load

The left graph of Figure 3 is the identified result in the case that a step-shaped distributed load with strength of 30 exists in the range of $4 \leq x \leq 6$. Similar to the previous example, though rather large overshoots occur near $x = 4$ and $x = 6$ where the value changes abruptly, it is a very good identification as a whole. It seems that to avoid this overshoot is impossible, yet the error can be restrained to a small enough range to meet with the needed accuracy in practical use. For example, we can re-arrange the source point over a narrower region where we suppose the load probably exist from the first calculation, or use the larger number of source point. The right graph of Figure 3 is the result when we use 99 point source over a range of $3 \leq x \leq 7$. Compared to the left one, the outlook of the step is identified more clearly and the overshoots near the step becomes smaller, too.

Figure 4 is the result for two distributed loads, namely, a step-shaped linearly distributed load expressed by $q(x) = -x + 13$ is applied in the range of $1 \leq x \leq 5$, and a step-shaped constant distributed load with the magnitude of 2 is applied in $6 \leq x \leq 9$. This shows that each distribution can be identified with the accuracy as good as that in the case when each distribution is applied individually.

As shown in the above, it is proved that the load can be identified accurately by the method of this study without any assumptions such as those of the kind of load, the number of load and applied position.

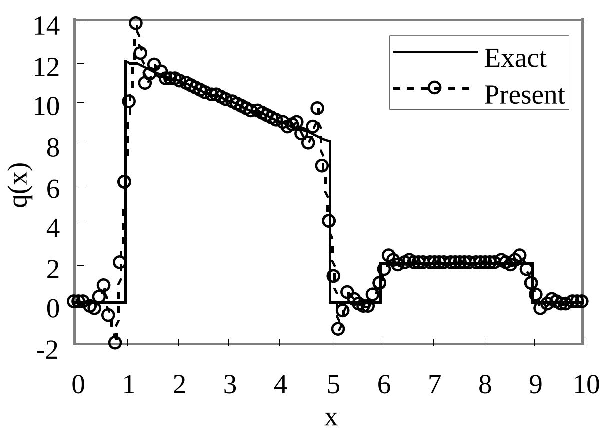


Figure 4: Identification of two distributed loads

2.3 *Influence of order n of Nabla-operator and other boundary conditions*

All of the above results are obtained using equation of $n = 2$. If it is done with the equation of $n = 4$, the accuracy will be better. However, there is not so much difference as a whole because the overshoot near a step cannot be eliminate and the accuracy in the case of $n = 2$ is sufficient enough.

In all mentioned examples, the simple support boundary condition is used. Figure 5 is a problem which a beam has a roller at $x = 0$ and be fixed at $x = L$, the load distribution is shown in the figure. The behavior of the results is almost as same as in the (a) of Figure 3. So we can say that the difference in boundary condition has no influence to the new method.

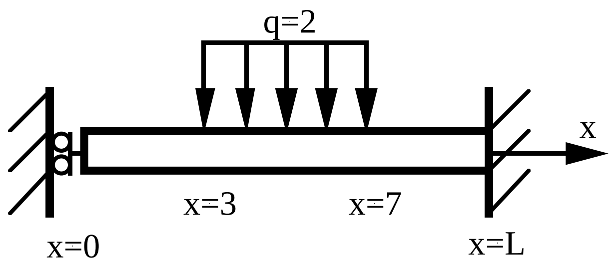


Figure 5: Analysis model with another boundary condition

CONCLUSIONS

In this paper, the discrete integral method utilizing the delta function was applied, and a new scheme to analyze the inverse problem using this method was demonstrated. In this report, identification problem of external load distribution on homogeneous beam was treated as an example of one dimensional inverse problem. By this analysis scheme, external load distribution can be identified accurately and naturally without any assumptions such as those of the kind of load, the number of load, and applied position. Further, the scheme is applicable to various boundary conditions. However, when the load to be identified is a concentrated one or has a steep change, the overshoot appears, and the errors apt to become larger round the edging point. It is possible to consider a practical scheme such as a re-identification using the result of the first identification to improve the accuracy. This analysis scheme can be expanded easily to a continuous beam.

The cases that ideal condition of identification can be instituted are treated in this paper. However, even for those problems with more complicated conditions, or of ill-conditions appeared in practical case, we consider that effective schemes can be developed based on the consideration of this analysis scheme.

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